Self-consistent screening in graphene

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Shaffique Adam
Yale-NUS College and
Center for advanced 2D materials
(formerly Graphene Research Center)
Department of Physics
National University of Singapore
Center for Advanced 2D materials (formerly Graphene Research Center)

Showing NUS physicists and collaborators… (not shown: chemists, engineers, people at NTU etc.)

**Theory colleagues**
- Antonio Castro Neto
- Giovanni Vignale
- Barbaros Özyilmaz

**Experimental colleagues**
- Baowen Li
- Feng Yuan Ping
- Zhang Chun
- Andrew Wee
- Barbaros Özyilmaz
- Su Ying Quek
- Vitor Pereira
- Sow Chorng Haur
- Lay-Lay Chua
- Hsin Lin
- Goki Eda
- Jens Martin
- Slaven Garaj
- Utkur Misaidov
- Peter Ho
- Andrew Wee
- Ji Wei
- Jose Gomes
- Wei Chen
- Christian Nijhuis

shaffique.adam@yale-nus.edu.sg
Research Group

Postdoctoral Research Fellows

Jeil Jung
Mirco Milletari
Joao Rodrigues
Derek Ho

Graduate Student Researchers

Indra Yudhistira
Navneeth Ramakrishnan
Tang Ho Kin
Jia Ning Leaw
The perfect 2DEG

Graphene is all surface and no bulk

“God made the bulk; surfaces were invented by the devil”
– Wolfgang Pauli

Figure from G. Rutter
Delicate interplay between disorder, interactions and quantum effects

Most experiments are in the regime, where all these effects are relevant, but not dominant!

See e.g.
M. S. Fuhrer and S. Adam, *Nature*, news and views, 458 38 (2009); and

Image credits: Fuhrer, Geim, Westervelt,
Semi-classical picture

Disorder

Potential

Electron gas

Figure from M. Wayne
Unlike conventional 2DEGs, graphene remains metallic even for strong disorder.

M. Isichenko, Rev. Mod. Phys. (1992)
Dirac materials: interaction strength and density tuned independently

**Interactions**

\[ r_s = \frac{e^2}{\hbar v_F} \]

\[ r_s = \frac{e^2}{\hbar v_F \left( \kappa_1 + \kappa_2 \right)} \]

Fermi liquid away from DP

Weak Interactions = Screening

\[ \varepsilon(q) = 1 + \frac{2\pi e^2}{\kappa q} \Pi(q) \]

- Screening is “metallic” on distances larger than the Fermi wavelength
- Screening is like a dielectric “insulator” on shorter distances
- Long-range nature of coulomb tail can not be screened

\[ \varepsilon(q) = 1 + \frac{2\pi e^2}{\kappa q} \Pi(q) \]
What about the Dirac point?

For strong interactions, Monte Carlo reveals gapped phase (anti-ferromagnet)

Strong Interactions

F. Assaad (2012-2014)

Semi-metal to AFM phase transition consistent with SU(2)  
SU(2) Heisenberg Gross-Neveu universality class

For strong interactions, Monte Carlo reveals gapped phase (anti-ferromagnet)
Divergent Fermi velocity

For weak interactions, perturbation theory reveals diverging Fermi velocity

\[ \frac{v_{\text{eff}}}{v_0} = 1 + \frac{r_s}{4} \log \left[ \frac{\sim 1}{ka} \right] \]

1. Renormalization Group [e.g. Sachdev (1998) / Guinea (1997)]

The fixed point is free Dirac theory where interactions are “dangerously irrelevant”.

2. Diagrammatic perturbation approaches [e.g. Das Sarma et al. (2007)]

“marginal Fermi liquid”

3. Hubbard model on a honeycomb lattice (semi-metal to AFM Mott transition occurs at interaction strengths outside the experimental window) e.g. Sorella, Assaad, Katsnelson (2012-2014).

“qualitatively similar to a Fermi liquid”

4. Lattice Monte Carlo applied to Dirac fermions with momentum cut-off e.g. Drut and Lahde (2009-2014).

“chiral symmetry breaking insulating state for suspended graphene”
Perfect transmission

Quantum Interference

P-N junctions are completely transparent for direct incidence. Results in beam collimation.
Universal ballistic $\sigma_{\text{min}}$

Quantum Interference

Without disorder and without interactions, both Kubo and Landauer formalisms give a universal ballistic minimum conductivity at the Dirac point.

M. Katsnelson (2006); C. Beenakker et al. (2006)
Delicate interplay between disorder, interactions and quantum effects

Most experiments are in the regime, where all these effects are relevant, but not dominant!

Disorder

Electron Interactions

Quantum Interference
Scaling theory of localization

Graphene

Disorder + Quantum Interference

2DEG with SOC

Schrödinger 2DEG
No Anderson localization

Dirac point conductivity (numerically exact)

\[ \sigma \frac{4e^2}{h} \]


\[ \langle U(r)U(r') \rangle = K_0 \frac{(hv)^2}{2\pi^2} e^{-|r-r'|^2/2\xi^2}, \]

Self-consistent diffusive Boltzmann transport theory

Ballistic universal quantum theory
How does an inhomogeneous Fermi liquid screen external potentials?

Example of Thomas-Fermi screening:

\[ V_{\text{screened}} = \frac{V_{\text{bare}}}{\varepsilon(q)} \]
How does an inhomogeneous Fermi liquid screen external potentials?

Example of Thomas-Fermi screening:

\[
V_{\text{screened}} = \frac{V_{\text{bare}}}{\varepsilon[q]}
\]

Regular 2DEG: \(\epsilon(q) = 1 + \frac{2e^2}{\hbar^2} \frac{m_e}{q}\)
How does an inhomogeneous Fermi liquid screen external potentials?

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Dirac electrons: \( \epsilon(q) = 1 + \frac{e^2}{\hbar v_F} \frac{\sqrt{\pi n}}{q} \)
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\( v_F \xrightarrow{2\text{DEG}} \frac{\hbar}{2m} \sqrt{\pi n} \)
How does an inhomogeneous Fermi liquid screen external potentials?

Example of Thomas-Fermi screening:

\[
V_{\text{screened}} = \frac{V_{\text{bare}}}{\varepsilon[q]}
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Regular 2DEG: \( \epsilon(q) = 1 + \frac{2e^2}{\hbar^2} \frac{m_e}{q} \)

Dirac electrons: \( \epsilon(q) = 1 + \frac{e^2}{\hbar v_F} \sqrt{\pi n} \frac{1}{q} \)

Inhomogeneous Dirac: \( \epsilon(q) = 1 + \frac{e^2}{\hbar v_F} \frac{\sqrt{\pi n_{\text{rms}}}}{\sqrt{3q}} \)
What is \( n_{\text{rms}}(n_{\text{imp}}) \)?

Statistical properties of density fluctuations and the Dirac point

Any physical observable could then be calculated

250 nm x 250 nm

1. Histogram of the carrier density (distribution function):

\[
P[n] \propto n_{\text{rms}}
\]

2. Screened potential correlation function:

\[
\langle \langle V(r)V(0) \rangle \rangle \propto \xi
\]

Characterizes the puddle depth and length

\[
\langle \langle V^2(0) \rangle \rangle
\]
What is \( n_{\text{rms}}(n_{\text{imp}}) \)?

**Statistical properties of density fluctuations and the Dirac point**

Any physical observable could then be calculated.

Calculate dimensionless quantity:

\[
\frac{n_{\text{rms}}}{n_{\text{imp}}} = ?
\]

1. Histogram of the carrier density (distribution function):

\[ P[n] \]

2. Screened potential correlation function:

\[ \langle \langle V(r)V(0) \rangle \rangle \]

\[ \langle \langle V^2(0) \rangle \rangle \]

Characterizes the puddle depth and length.
Ansatz for Inhomogeneous screening

\[ \phi_{\text{scr}}(q) = \frac{\phi_{\text{bare}}(q)}{\varepsilon(q, r_s, n)} \]

\[ \varepsilon(q) = \text{const} \quad \text{(dielectric)} \]

\[ \varepsilon(q) = 1 + \frac{q_{\text{TF}}}{q} \quad \text{(metal)} \]

\[ \varepsilon(q, k_F, r_s) = \begin{cases} 
1 + \frac{r_s \pi}{2} ; & q \ll 2k_F \\
1 + \frac{4k_F r_s}{q} ; & q \gg 2k_F 
\end{cases} \quad \text{(Dirac)} \]
Ansatz for Inhomogeneous screening

\[ \phi_{scr}(q) = \frac{\phi_{bare}(q)}{\varepsilon(q, r_s, \langle n \rangle = 0)} \]

\[ \varepsilon(q, r_s, P[n]) \rightarrow \varepsilon(q, r_s, \langle n^2 \rangle, \langle n^3 \rangle, \ldots) \]
Ansatz for Inhomogeneous screening

\[ \phi_{scr}(q) = \frac{\phi_{bare}(q)}{\frac{\epsilon(q, r_s, \langle n \rangle = 0)}{\epsilon(q, r_s, \langle n \rangle = 0)}} \]

\[ \epsilon(q, r_s, P[n]) \rightarrow \epsilon(q, r_s, \langle n^2 \rangle, \langle \ldots \rangle) \]
Ansatz for Inhomogeneous screening

\[ \phi_{scr}(q) \approx \frac{\phi_{bare}(q)}{\varepsilon(q, r_s, n_{rms})} \]

\[ \varepsilon(q, r_s, P[n]) \approx \varepsilon(q, r_s, \langle n^2 \rangle) \]
Ansatz for Inhomogeneous screening

\[ \phi_{scr}(q) \approx \frac{\phi_{bare}(q)}{\varepsilon(q, r_s, n_{rms})} \]

\[ \frac{n_{rms}}{n_{imp}} = 2 r_s^2 C_0 \left[ r_s, d \sqrt{n_{rms}} \right] \]

S. Adam, E. H. Hwang, V. M. Galitski and S. Das Sarma
Local screening vs. global screening

E. Rossi and S. Das Sarma
PRL 101, 166803 (2008)


$$V_{\text{screened}} = \frac{V_{\text{bare}}}{\varepsilon[q, n_{\text{rms}}]}$$

$$x \sim q/\sqrt{n_{\text{rms}}}$$
Numerical verification

\[ n_{\text{rms}} \]

S. Adam, E. H. Hwang, V. M. Galitski and S. Das Sarma

E. Rossi, S. Adam and S. Das Sarma
Puddle Formation

With J. A. Stroscio (NIST)

Single Layer Graphene

Bilayer Graphene

Electron puddle

Electron puddle

Hole puddle

Hole puddle

20 nm

20 nm
Agreement with experiments [1]

Collaboration with J. A. Stroscio (NIST)

Single Layer Graphene

Bilayer Graphene

No adjustable parameter!
Monolayer graphene from a Brian LeRoy (Arizona)

Bilayer Graphene

Agreement with experiments [1]

Collaboration with J. A. Stroscio (NIST)

No adjustable parameter!