An exotic quantum phase transition with Dirac fermions

Shailesh Chandrasekharan
(Duke University)

Work done in collaboration with
Venkitesh Ayyar
(related work arXiv: 1410.6474)

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Introduction

- Relativistic four-fermion and Yukawa models have a long history.
- Can be studied using lattice field theories.
  - was popular in the late eighties.
  - the model we discuss here was studied long ago!
- Many calculations of fermionic critical points in the mid nineties.
  - studies use HMC but add a small fermion mass.
  - most calculations on lattices about 40^3 or less.
  - sign problems hinder applications.
  - critical exponents not usually measured accurately.

- Traditional (HMC) algorithms involve inverses of (singular) matrices whose size is the space-time volume. Blankenbecler, Scalapino, Sugar, Scalettar, Duane, Kennedy, Pendleton, ...

- Recently, diagrammatic Monte Carlo methods have become popular. Rubtsov, Savkin, Lichtenstein, Prokof'ev, Svistunov, Gull ,...

- Fermion bag approach: a generalization of this idea applied to lattice field theories. S.C. (2008), PRD82 (2010)

- Fermion bags give further insight into diagrams
  - duality between weak and strong couplings.
    S.C and Li PRL 108 (2012), S.C. EPJA49 (2013)
  - some old sign problems become solvable.
    S.C. PRD86, (2012); Huffman and S.C PRB89, (2014);
  - ideas for speeding up fermion algorithms.
    S.C. and Ayyar, in preparation

Highlights of Fermion Bag Approach

Highlights	Traditional Approach Debbio & Hands (1997), Barbour et al (1998)	Fermion Bag Approach S.C & A. Li (2012, 2013)
Calculations with exactly massless fermions	$m = 0.025 - 0.005$	$m = 0$
Largest volumes possible	$12^3 - 40^3$	$12^3 - 40^3$
Critical exponents calculated about 10 times more accurately.	$\nu = 0.80(15)$, $\eta = 0.40(20), 0.70(15)$	$\nu = 0.849(8)$ $\eta = 0.633(8)$ $\eta_\psi = 0.373(3)$

Goal of this talk:

Discuss how the fermion bag approach
allows us to solve another old, but interesting
lattice field theory model
with an exotic quantum transition
which may be of interest in CM physics.

Model

Action on a cubic space-time lattice:

4d models: Hasenfratz, Neuhaus, Lee, Shigemitsu, Shrock, Smit, De,Bock …. (1987-1992)

$$S = \frac{1}{2}\sum_{x,\alpha,i=1,2} \eta_{x,\alpha}\left(\overline{\psi}_{x,i}\psi_{x+\alpha,i} - \overline{\psi}_{x+\alpha,i}\psi_{x,i}\right) - U\sum_{x}\overline{\psi}_{x,1}\psi_{x,1}\overline{\psi}_{x,2}\psi_{x,2}$$

Partition function:

$$Z = \int [d\overline{\psi}\ d\psi]\ \mathrm{e}^{-S}$$

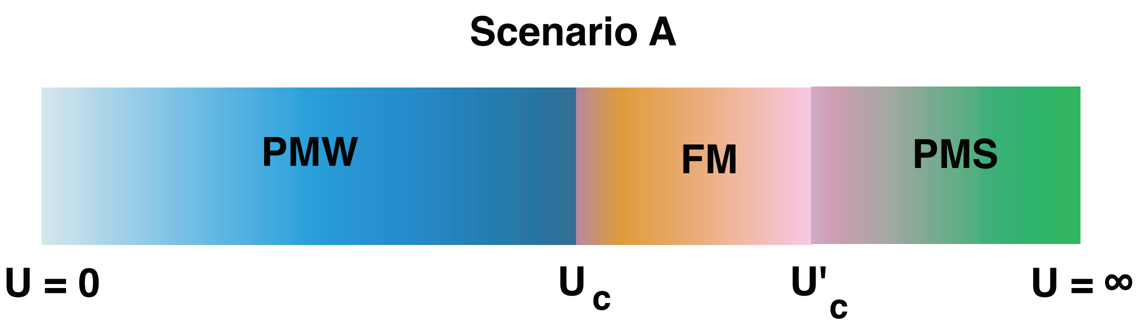
Observables:

$$\langle O\rangle = \frac{1}{Z}\int [d\overline{\psi}\ d\psi]\ \mathrm{e}^{-S}\ O(\overline{\psi},\psi)$$

Symmetries: Translations, Rotations, Reflection, SU(4)

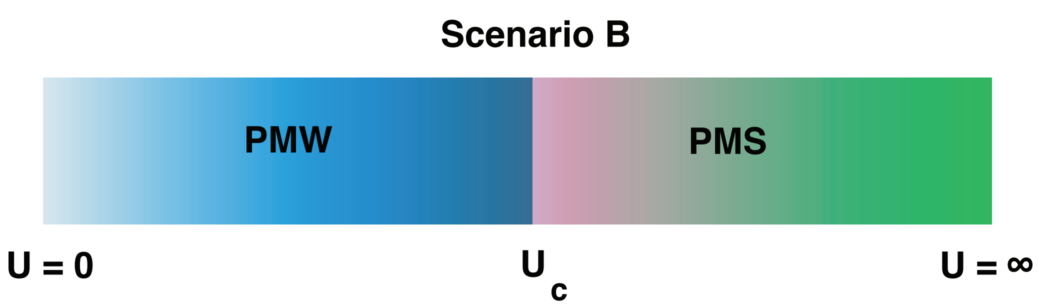
Fermion bilinear condensates necessarily break the SU(4) symmetry.

Phase Diagram



Traditional scenario

Stephanov, Tsypin (1990), Ebihara, Kondo (1992)



Exotic scenario:
A single second order transition

Slagle, You and Xu (2014)

Ayyar and S.C. arXiv:1410.6474

PMW

Weak paramegnetic phase
(massless fermions)

FM

Ferromagnetic phase
(massive fermions)
(with a fermion-bilinear condensate)

PMS

Strong paramegnetic phase
(massive fermions,
no fermion-bilinear condensate)

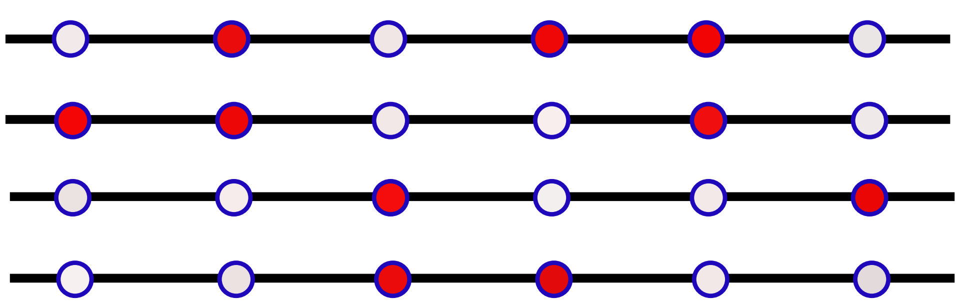
Honeycomb Bilayer Model

Slagle, You and Xu (2014)

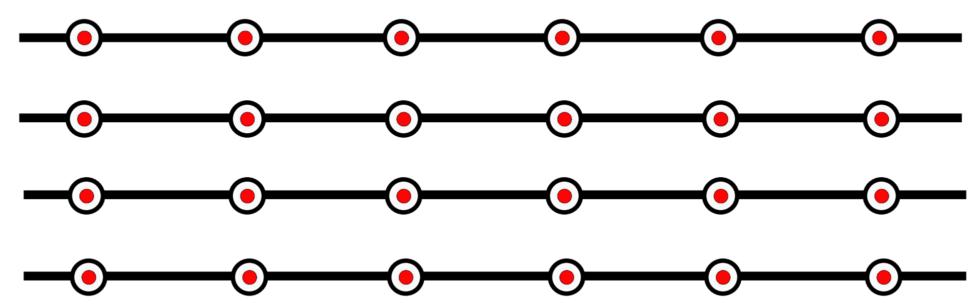
CM system with similar physics: $H = H_0 + U \; H_{\text{int}}$

$$H_0 = -t \sum_{\langle ij \rangle, \sigma=\uparrow,\downarrow, \ell=1,2} \left(c^\dagger_{i,\ell,\sigma} c_{j,\ell,\sigma} + c^\dagger_{j,\ell,\sigma} c_{i,\ell,\sigma} \right)$$

$$H_{\text{int}} = -U \; |\psi\rangle\langle\psi| \qquad |\Psi\rangle = \frac{1}{\sqrt{2}} \left(1 - c^\dagger_1 c^\dagger_2 c^\dagger_3 c^\dagger_4\right) |0\rangle$$



U = 0 (free massless fermions)



U = ∞ (onsite singlets)

Fermion Bags

Instead of using the HS transformation expand in the interaction strength:

$$Z = \int [d\overline{\psi}\ d\psi]\ \mathrm{e}^{-S_0} \prod_x \left(1\ +\ U\ \overline{\psi}_{x,1}\psi_{x,1}\overline{\psi}_{x,2}\psi_{x,2}\right)$$

$$Z = \sum_{[n]} U^k \int [d\overline{\psi}\ d\psi]\ \mathrm{e}^{-S_0}\ \overline{\psi}_{x_1,1}\psi_{x_1,1}\overline{\psi}_{x_1,2}\psi_{x_1,2}\ \ldots\ \overline{\psi}_{x_k,1}\psi_{x_k,1}\overline{\psi}_{x_k,2}\psi_{x_k,2}$$

k = number of monomers

There are two different ways of performing the Grassmann integral.

$$Z = \sum_{[n]} U^k \int [d\overline{\psi}\ d\psi]\ e^{-S_0}\ \overline{\psi}_{x_1,1}\psi_{x_1,1}\overline{\psi}_{x_1,2}\psi_{x_1,2}\ \ldots\ \overline{\psi}_{x_k,1}\psi_{x_k,1}\overline{\psi}_{x_k,2}\psi_{x_k,2}$$

Weak coupling approach:

$$Z \ = \ \mathrm{Det}(D) \sum_{[n]} U^k\ \mathrm{Det}(G)$$

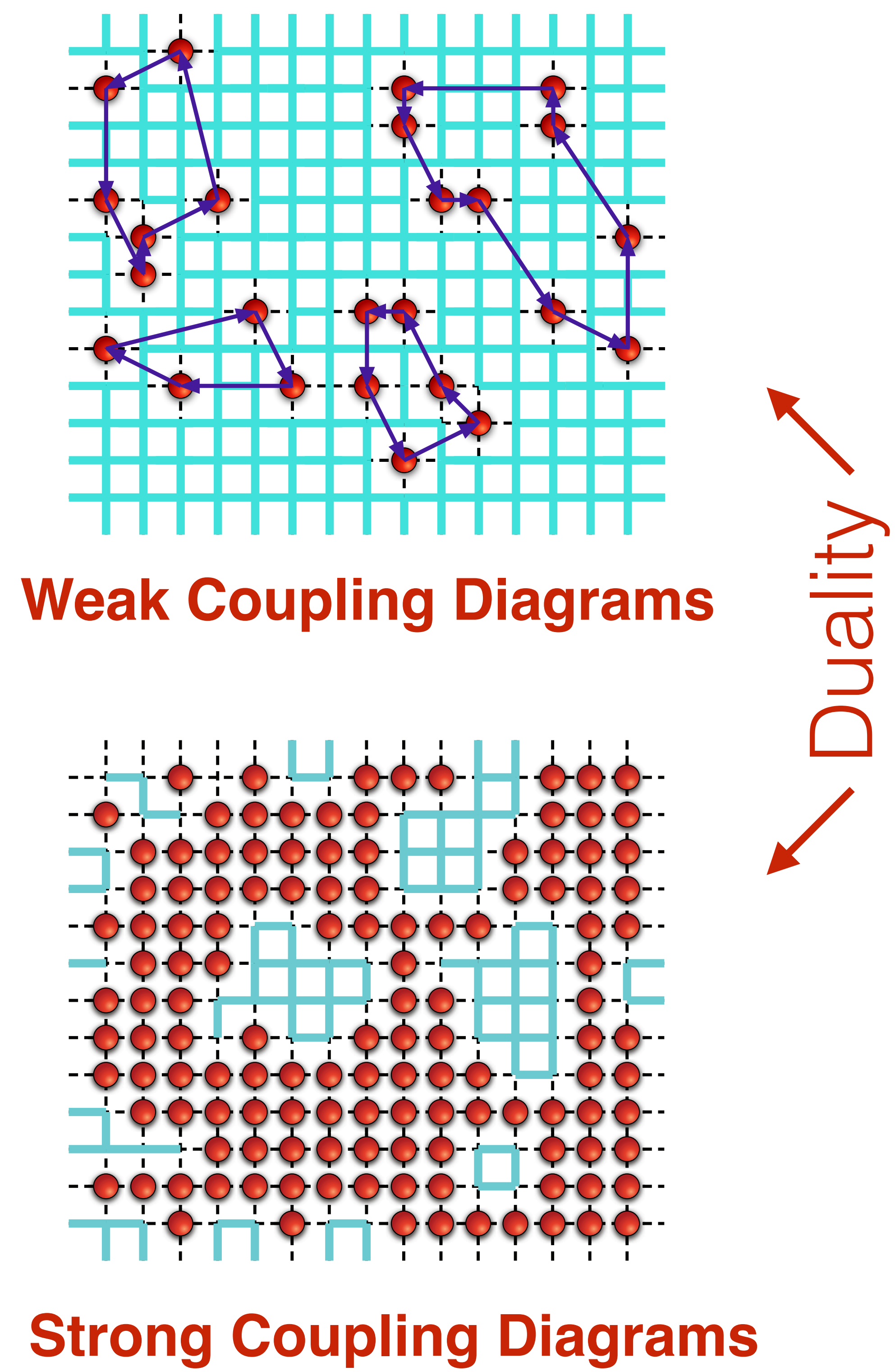
G is a k x k matrix

Strong coupling approach:

$$Z \ = \sum_{[n]} U^k\ \mathrm{Det}(W)$$

W is a (V-k) x (V-k) matrix

space-time splits into regions that look like "bags" inside which fermions propagate.



In general we can view both approaches as the “fermion bag approach”

Review Article: S.C. EPJA49 (2013)

A fermion bag refers to a group of fermion degrees of freedom.

In the fermion bag approach the full partition function is written as a sum over configurations of fermion bags.

If we can make sure that a resummation (trace) over the degrees of freedom inside a fermion bag yields positive numbers then there are no sign problems.

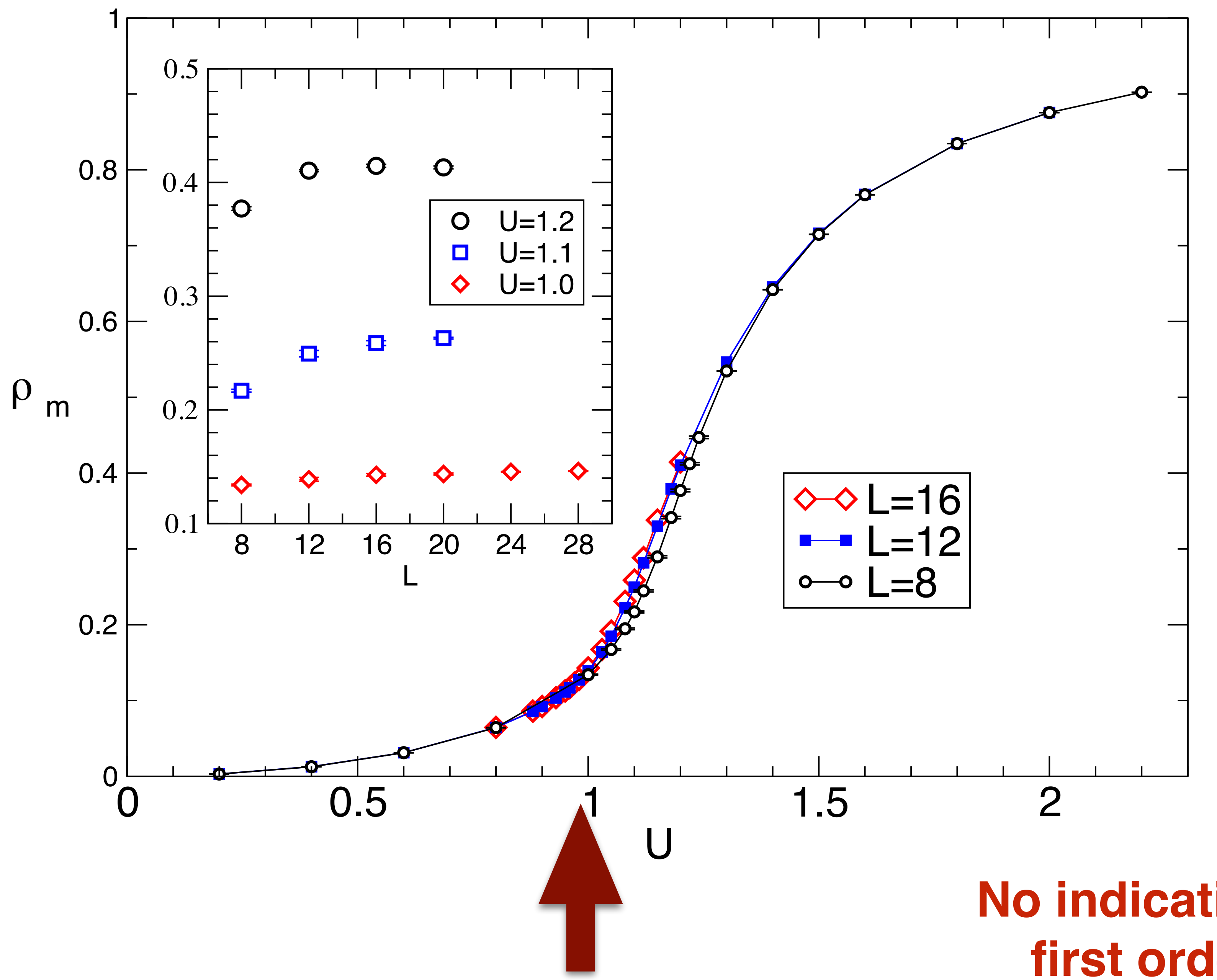
Some unsolved sign problems can be solved using this idea.

S.C. PRD86, (2012); Huffman and S.C PRB89, (2014);

Can also be used to accelerate fermion algorithms

S.C. and Ayyar, in preparation

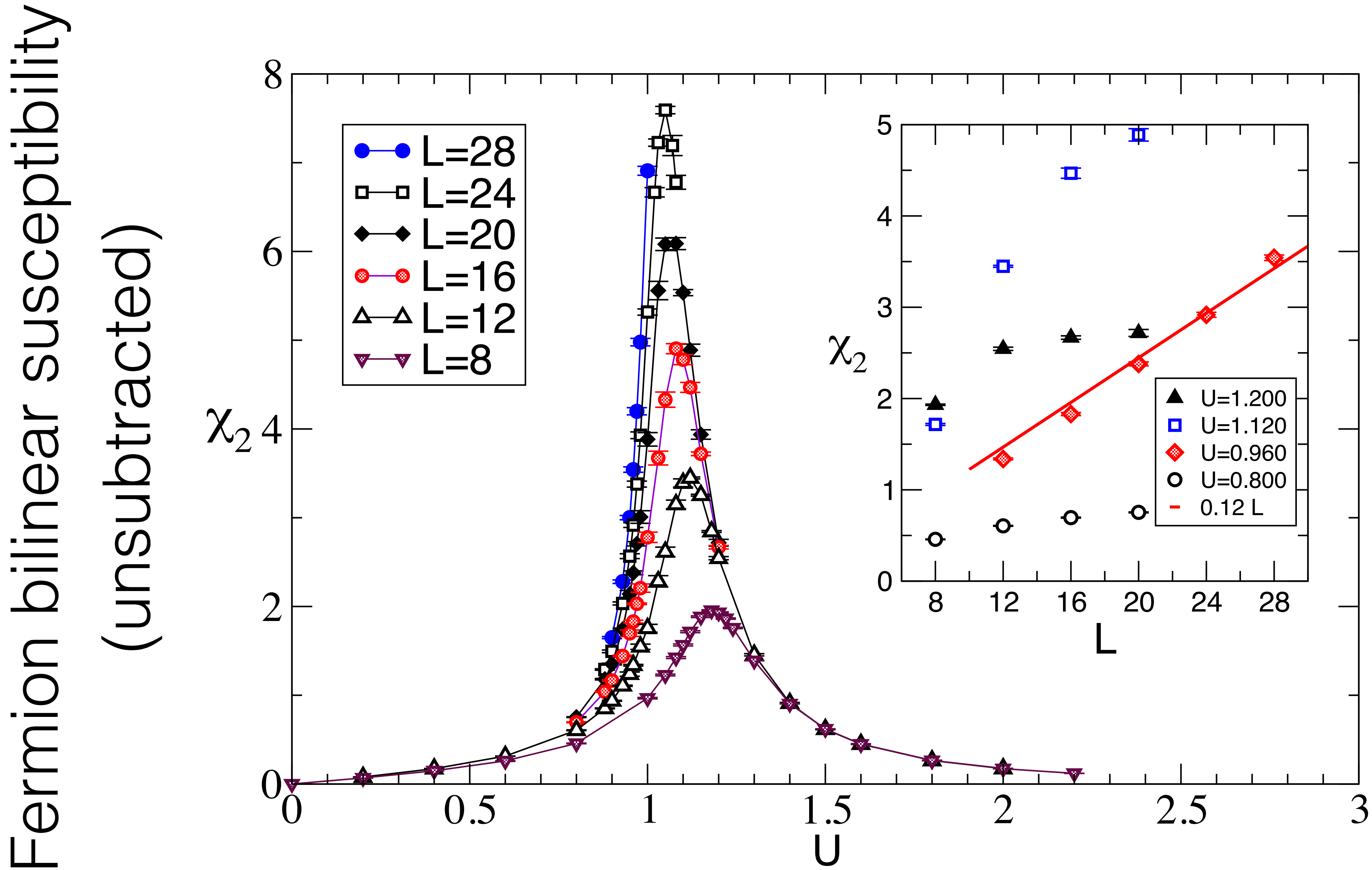
Monomer Density



No indication of a strong first order transition

critical point ≈ 0.96

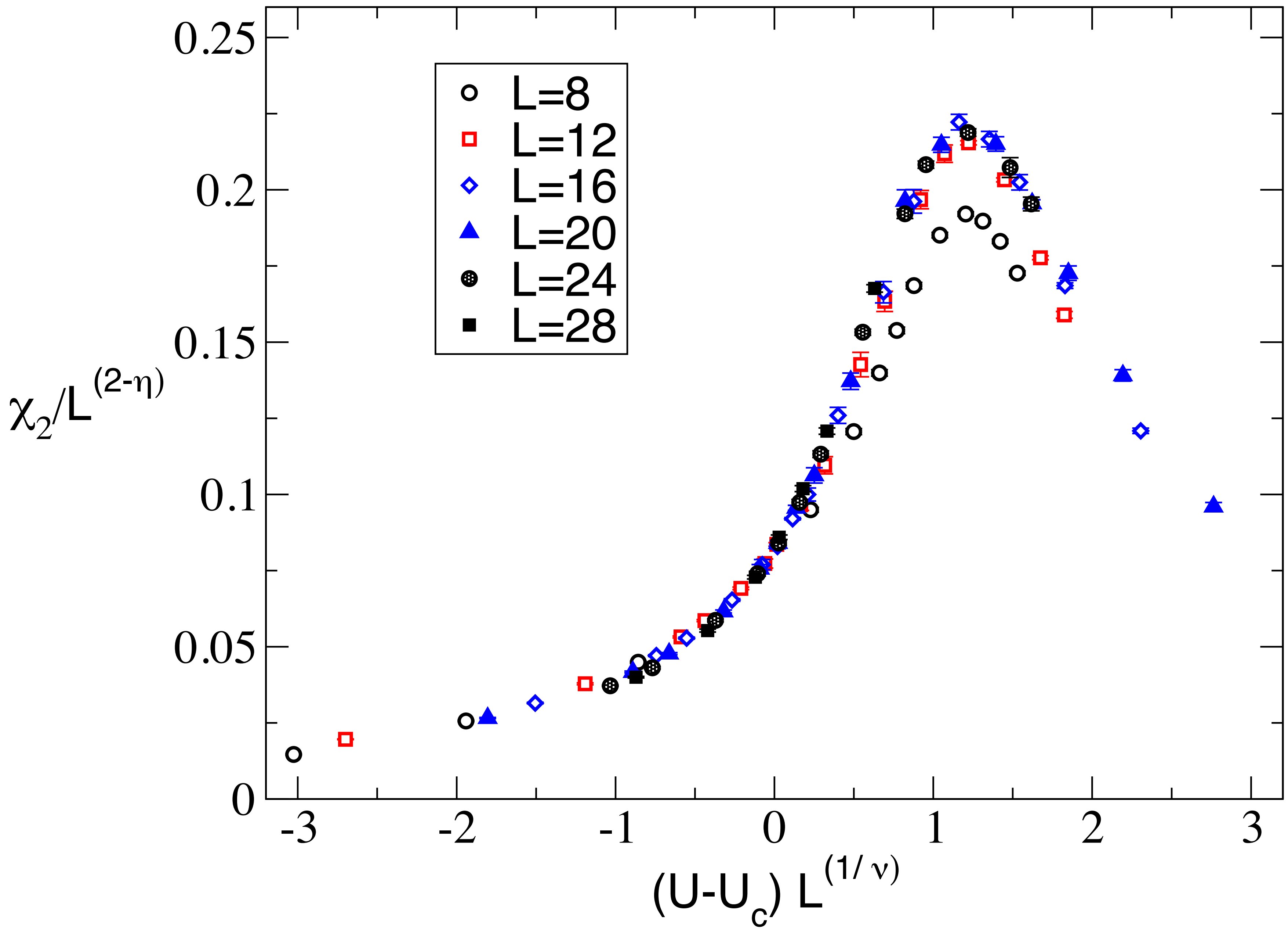
Susceptibility



more details:
Ayyar and S.C. arXiv:1410.6474

In the presence of a non-zero fermion bilinear condensate we expect $\chi_2 \sim L^3$.

Critical Exponents



Estimates

η : 0.88 – 0.94

ν : 0.95 – 1.2

Larger lattices are necessary to compute critical exponents more accurately

Fluctuations as fermion bags

Fluctuations of a background configuration
can naturally define fermion bags

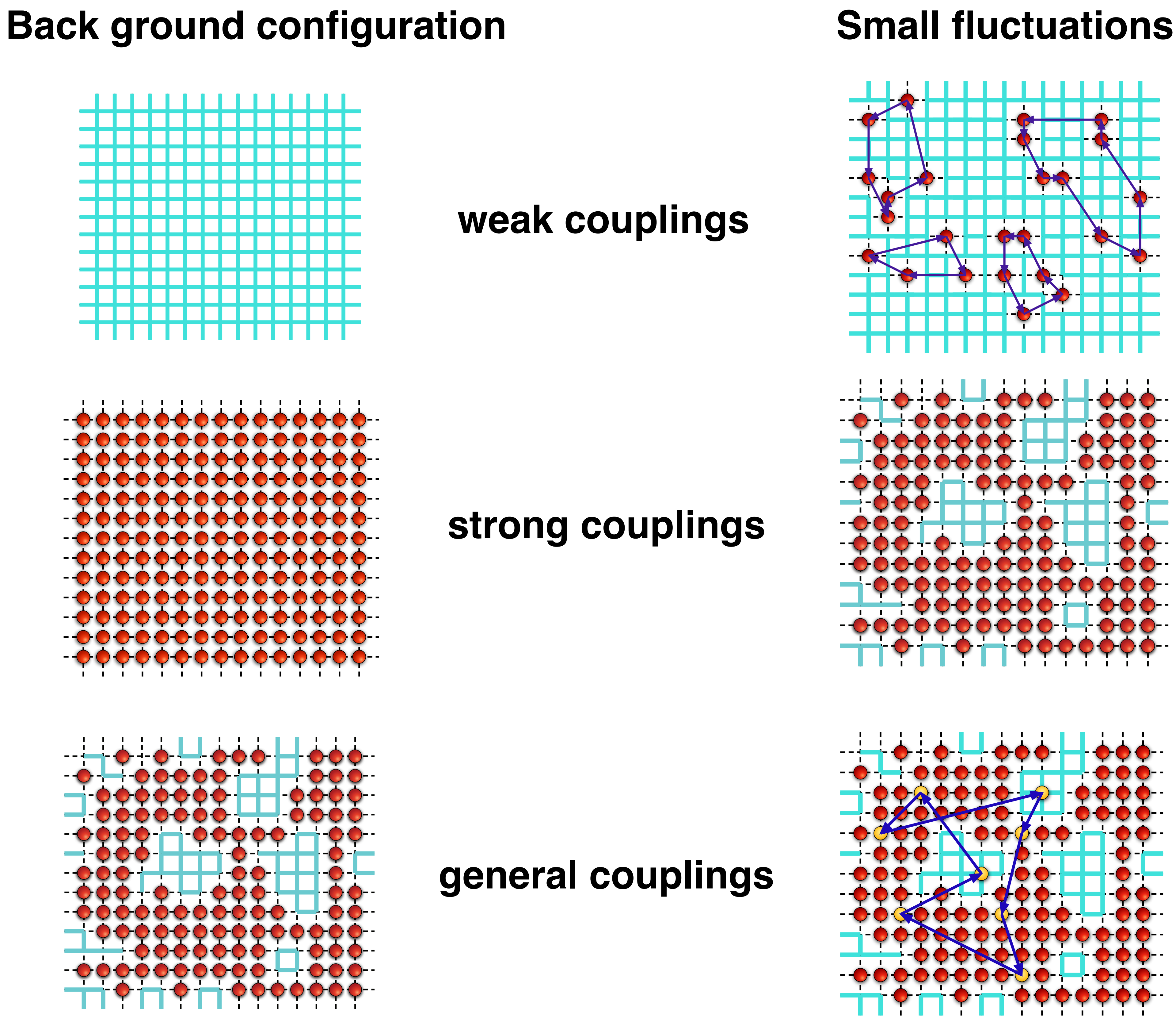
$$Z = \mathrm{Det}(A_{\mathrm{background}}) \sum_{[n_f]} U^k \ \mathrm{Det}(\Delta_{\mathrm{fluctuation}})$$

$k \times k$ matrix

Here we assume some background configuration of monomers is given.

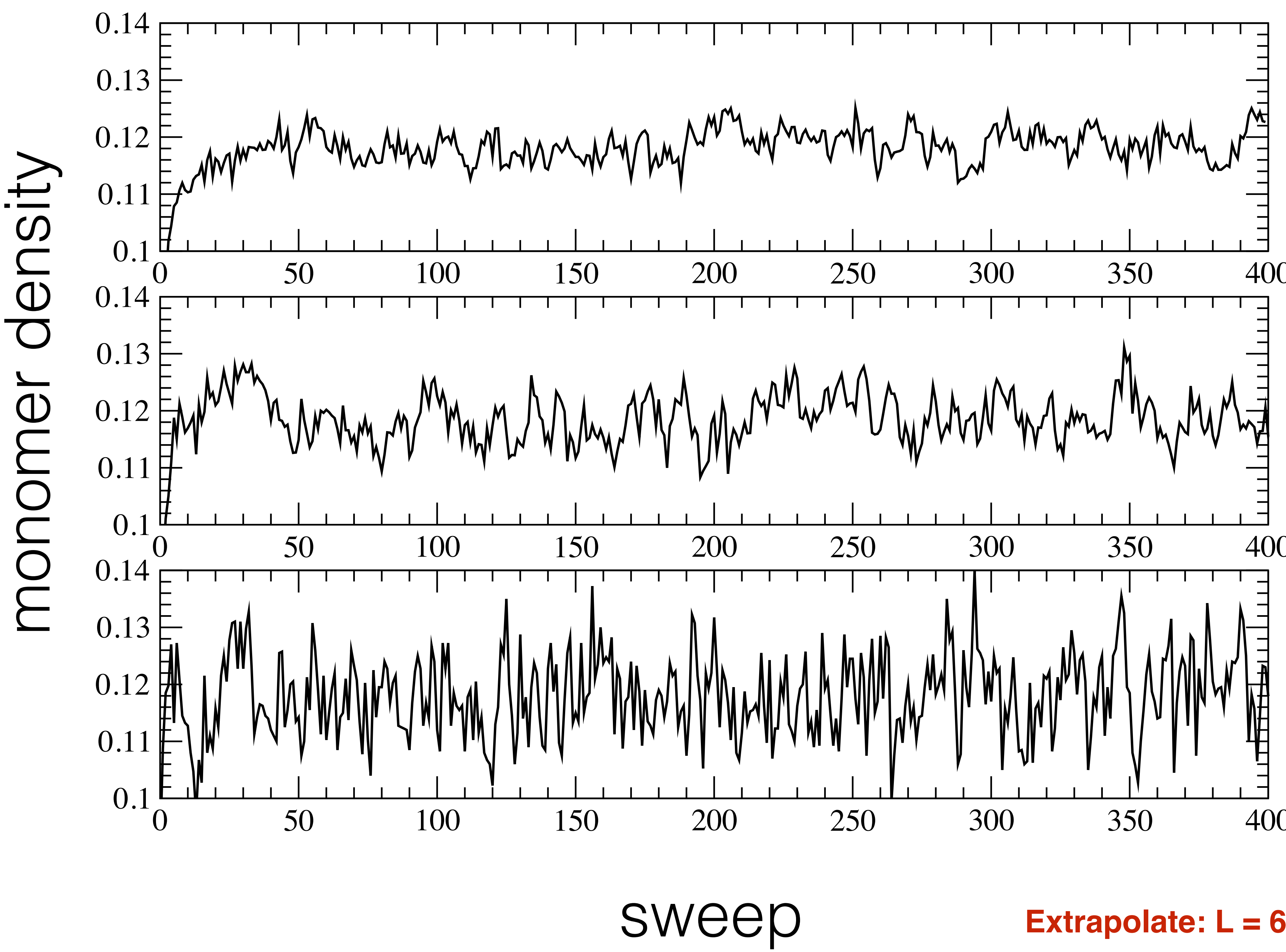
$[n_f]$ represents the fluctuation with k sites different from the background.

When k is small
this idea can be used to speed up the calculations!



Back ground configuration
Small fluctuations
weak couplings
strong couplings
general couplings

Monte Carlo Evolution



L = 40
1030 secs/sweep/core

L = 32
360 secs/sweep/core

L = 20
130 secs/sweep/core

Extrapolate: L = 60, 4hrs/sweep/core

Conclusions

- Fermion bags (or equivalently diagrammatic Monte Carlo methods) seem ideally suited to study a variety of strongly correlated fermion models.
- There is a simple 3d lattice model which shows an exotic quantum phase transition between a massless and a massive fermion phase. No symmetry breaking of either side!
- Physics of topology? What about 4d?
- The phase transition should be in the same universality class as the one recently proposed in a model of Dirac fermions on a bilayer honeycomb lattice.