Detecting signatures of topological order

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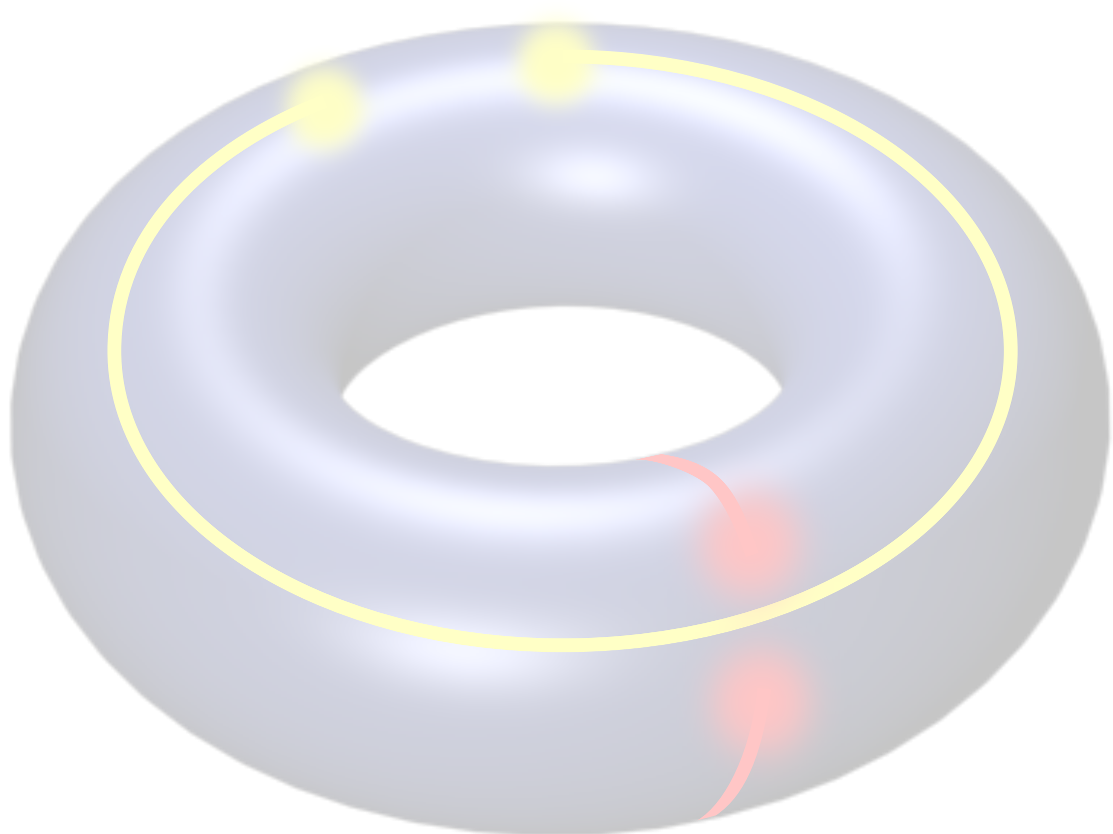
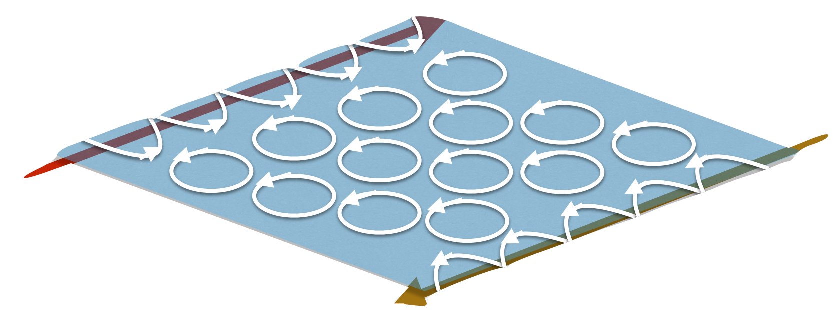


Würzburg. 24. Feb. 2015

Detecting signatures of topological order

- In the last years several **"topological" phases** have been discovered that **cannot be described by symmetry breaking**
  - Quantum Hall effects,
    [Klitzing '80, Tsui '82, Laughlin '83]
  - Topological Spin Liquids
    [Anderson '73]

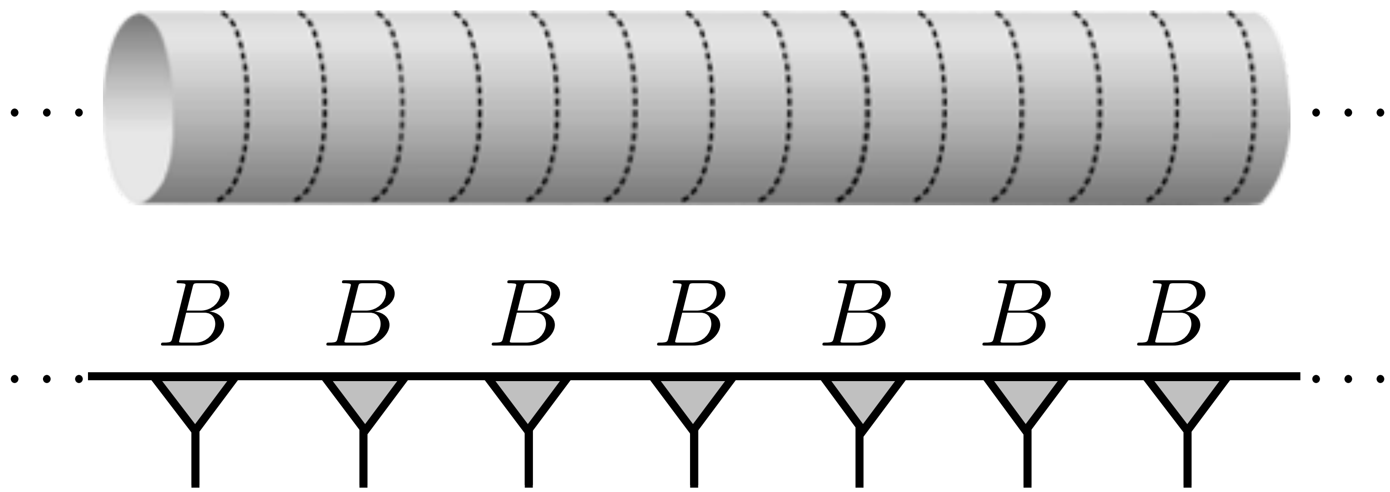


- **Fractionalization, "anyonic" quasiparticles:**
  Fault tolerant quantum computing (?) [Kitaev '03]

- **Search for physical systems that stabilize topological orders**

Detecting signatures of topological order

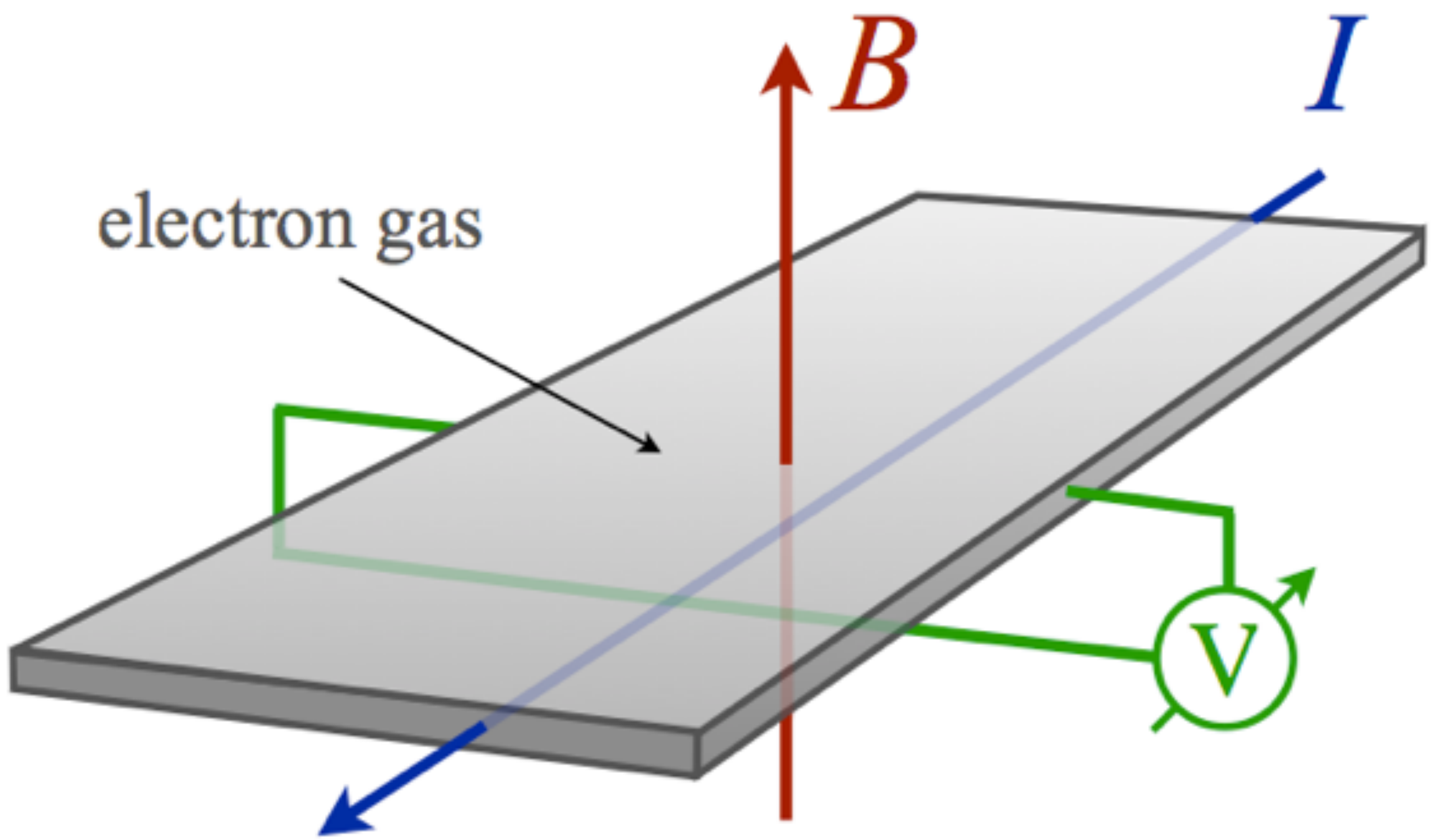
- Simulation of two-dimensional systems using the cylinder geometry: **infinite Density Matrix Renormalization Group (iDMRG)** [White '92, McCulloch '07]



➡ Efficient variational calculation of the ground state

➡ Extract characteristic fingerprints of topological order

Topological Order: Fractional Quantum Hall

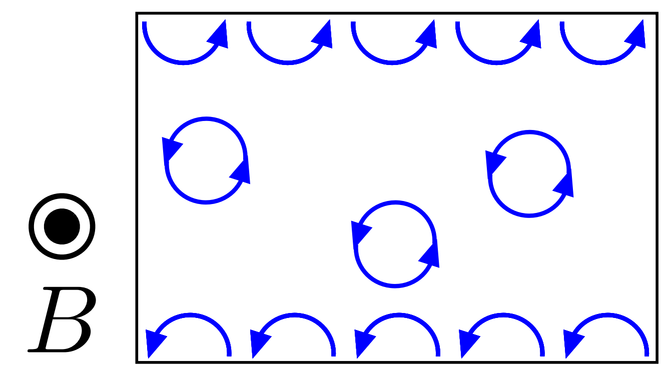


[Klitzing ’80, Tsui, Stormer ’82, Laughlin ’83]

Topological Order: Fractional Quantum Hall

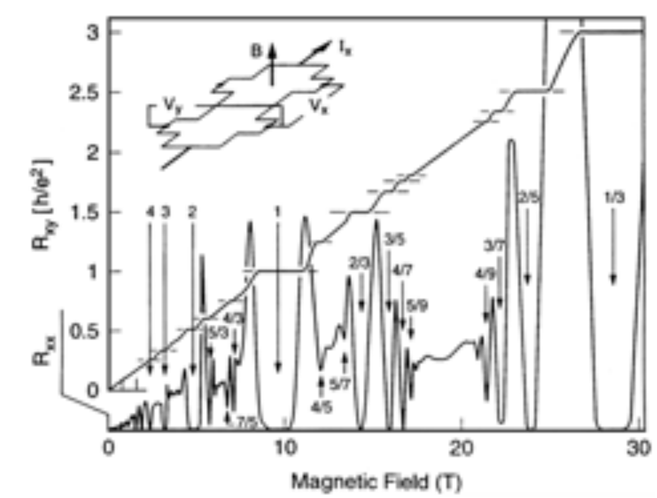
- **2D electron gas in magnetic field** B :
  highly degenerate "Landau levels"

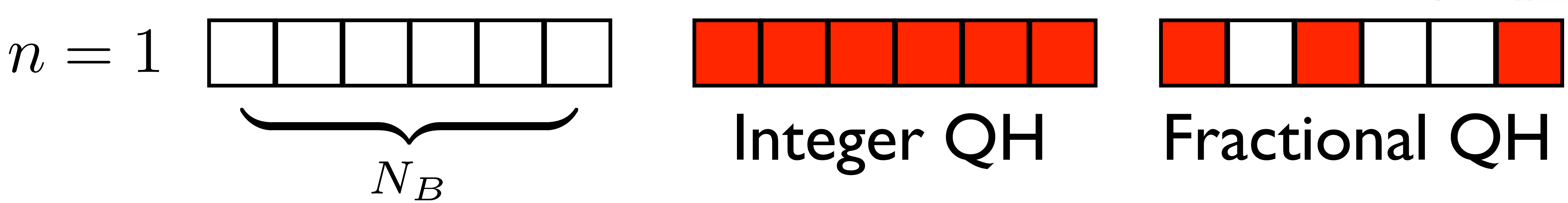
$$E_n = h\frac{eB}{m}(n+\frac{1}{2})$$



- Number N_B of degenerate orbits in each Landau level equal to number of flux quanta: **Filling fraction** $\nu = N_e/N_B$

- Incompressible liquid at integer fillings
  [Klitzing '80]
- **Fractional quantum Hall effect (FQHE):**
  Incompressible liquid due to **interactions**
  [Tsui, Stormer '82, Laughlin '83]

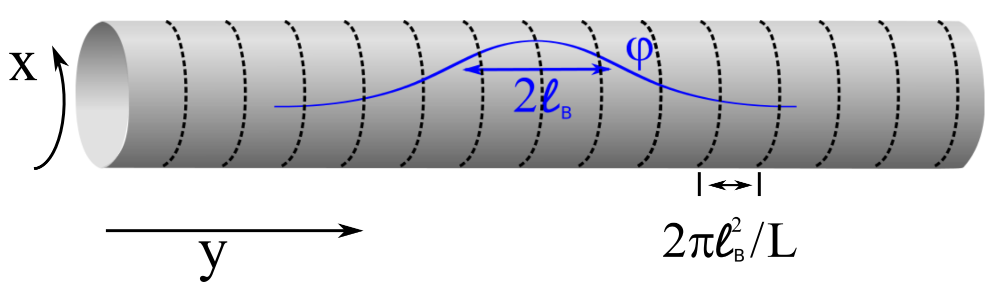


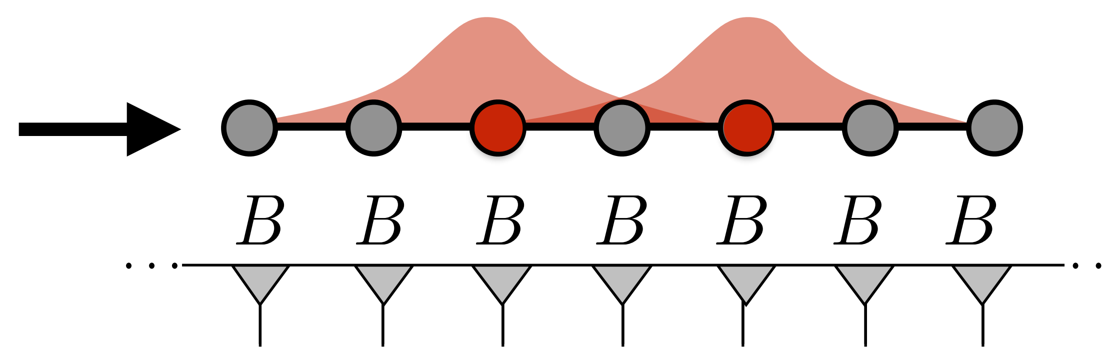


Topological Order: Fractional Quantum Hall

- Consider the **FQHE** on an **infinitely long cylinder**
  - Orbitals are localized along the cylinder: Quasi **1D model** using an occupation number basis $|\ldots, j_0, j_1, \ldots\rangle$

    [Haldane & Rezayi '94; Bergholtz et al. '05, Seidel et al. '05]





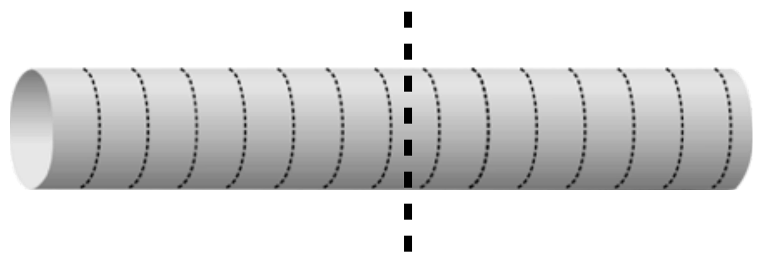
- **Infinite DMRG** allows for much larger system than accessible using exact diagonalization

- Conserve charge $\hat{C} = \sum_j \hat{N}_j$ and momentum $\hat{K} = \sum_j j\hat{N}_j$

M. P. Zaletel, R. S. K. Mong, FP, PRL **110**, 236801 (2013).

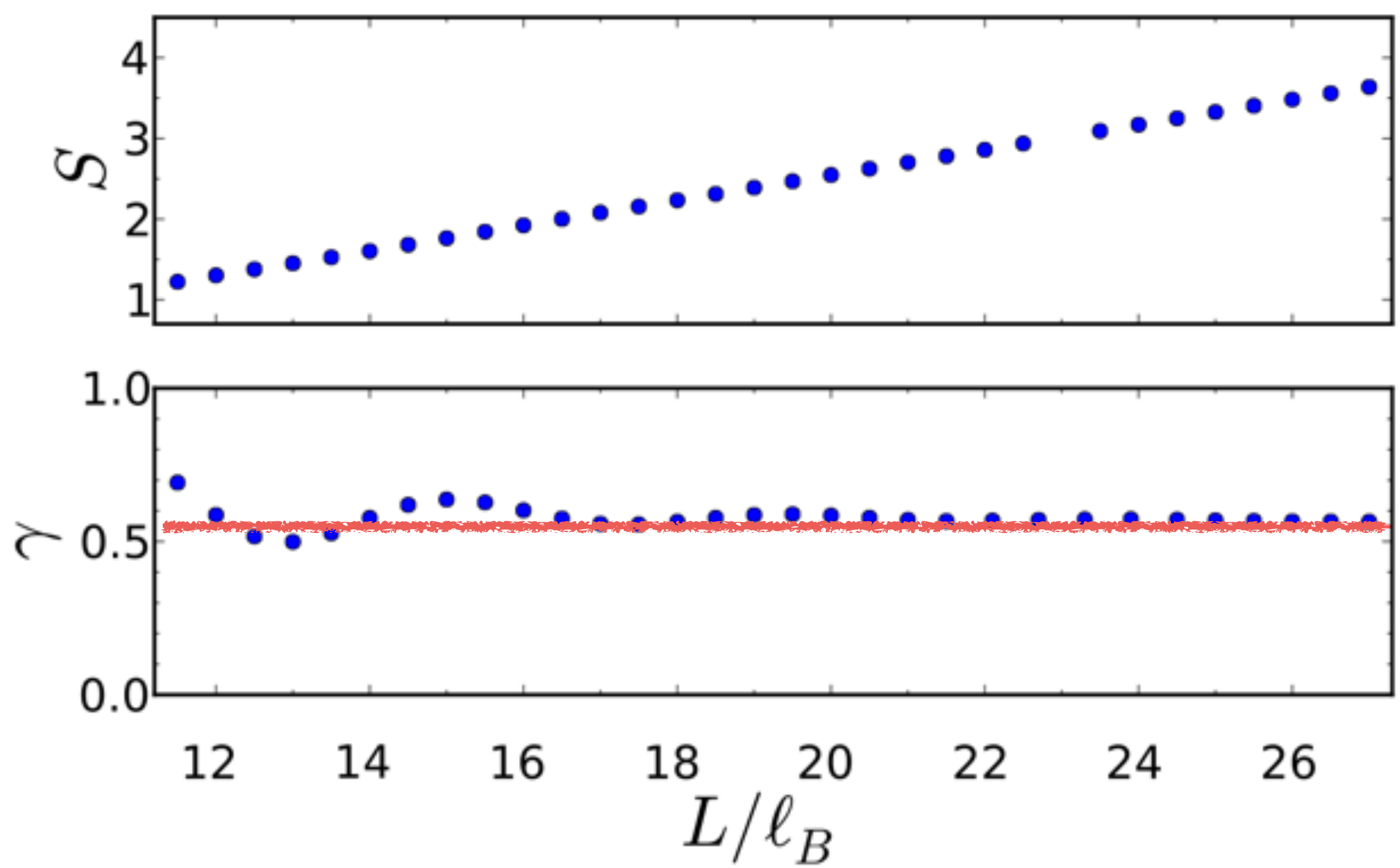
Topological Order: Fractional Quantum Hall

- **Topological entanglement entropy** of the $\nu = 1/3$ FQHE with Coulomb interactions



$$S = sL - \gamma_a$$

[Kitaev & Preskill '06]

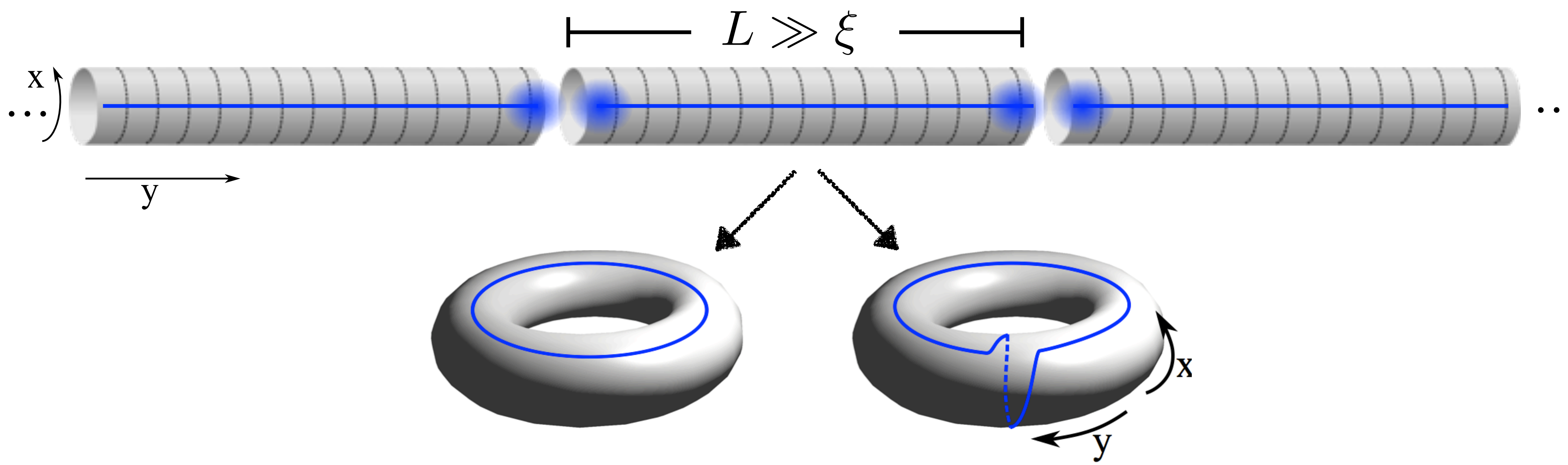


$\log \sqrt{3}$

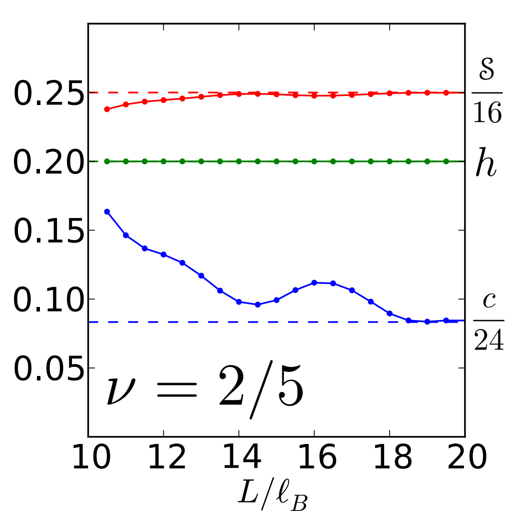
M. P. Zaletel, R. S. K. Mong, FP, arxiv:1410.3861 (2014).

Topological Order: Fractional Quantum Hall

- Extracting topological content by adding a “twist”



- Momentum polarization: **topological spin, central charge, Hall viscosity**



$$U_{T;ab} = \delta_{ab} \exp\left[2\pi i \left(h_a - \frac{c}{24} - \frac{\eta_H}{2\pi\hbar} L_x^2\right)\right]$$

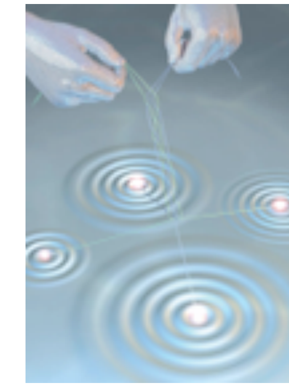
(see also Zhang et al. ’12, Tu et al. ’13, Cincio & Vidal ‘13)

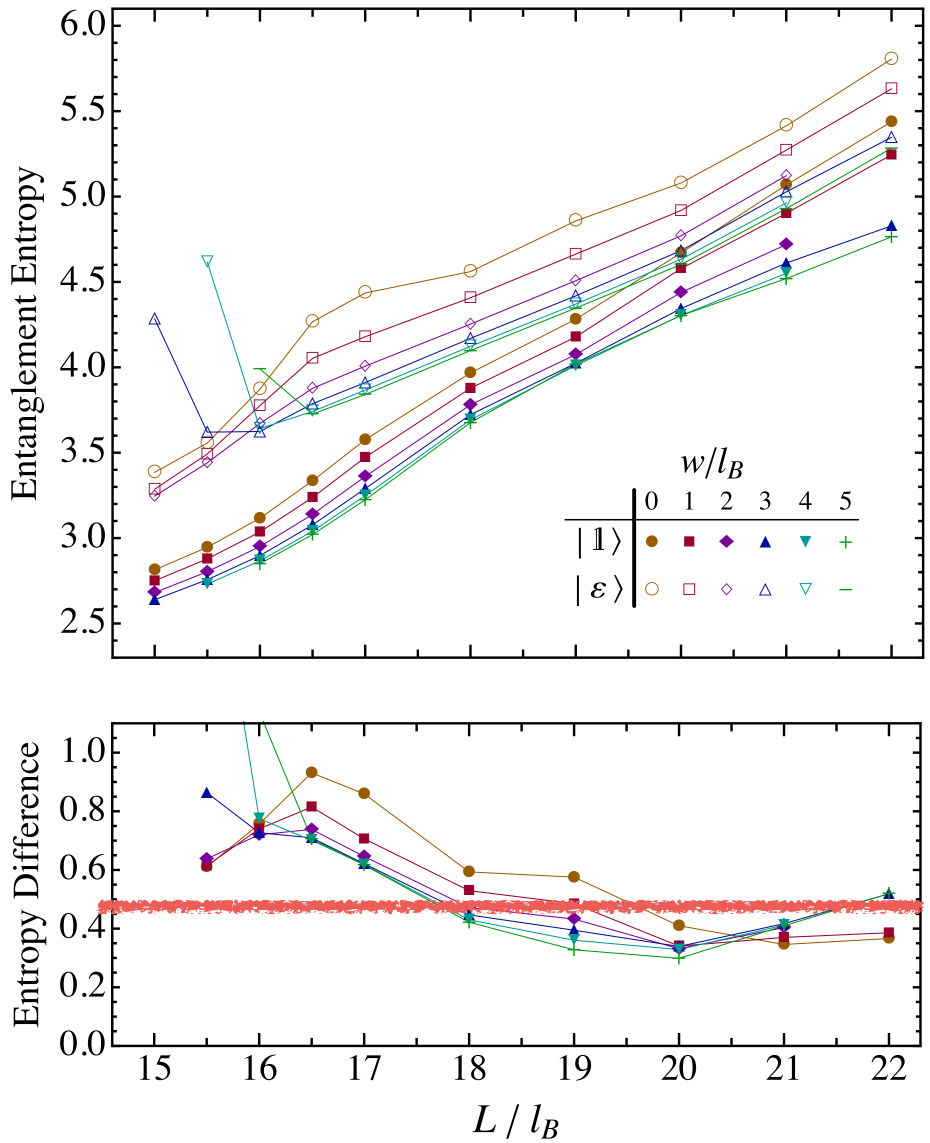
M. P. Zaletel, R. S. K. Mong, FP, J. Stat. Mech. P10007(2014).
M. P. Zaletel, R. S. K. Mong, FP, PRL **110**, 236801 (2013).

Topological Order: Fractional Quantum Hall

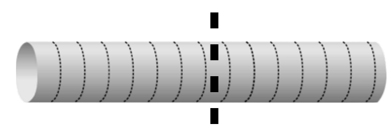
- iDMRG on FQHE at $\nu = 12/5$:
  Numerical evidence for the existence of Fibonacci anyons! [Read & Rezayi '98]

$$\varphi = \frac{1+\sqrt{5}}{2} \approx 1.61803$$

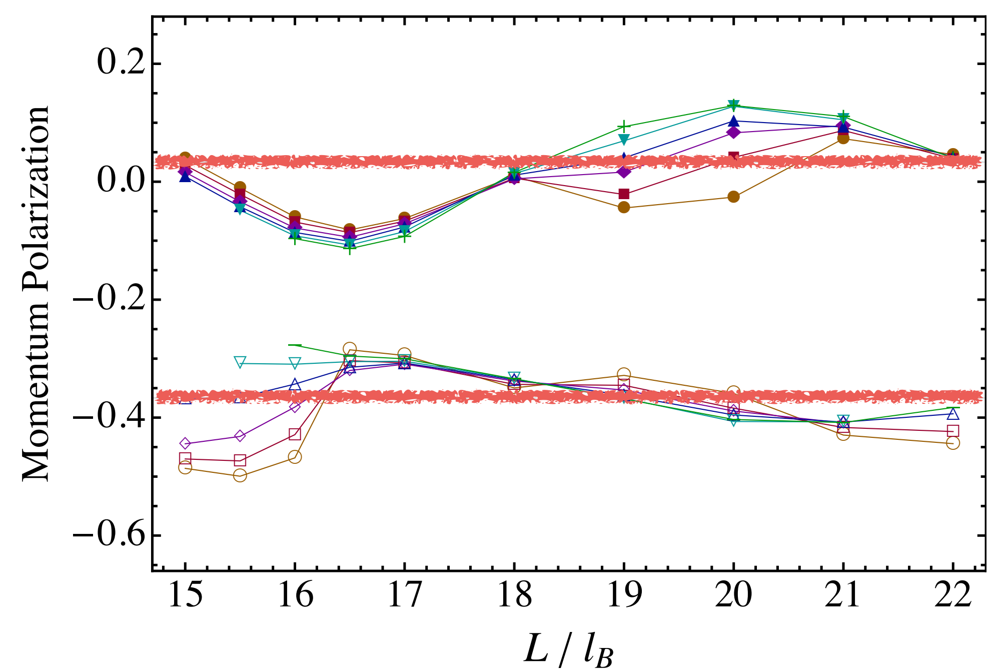


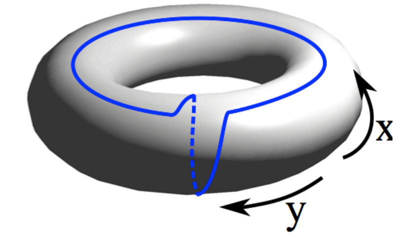


$$S = sL - \gamma_a \qquad \gamma_a = -\log\left(d_a \Big/ \sqrt{\sum_b d_b^2}\right)$$



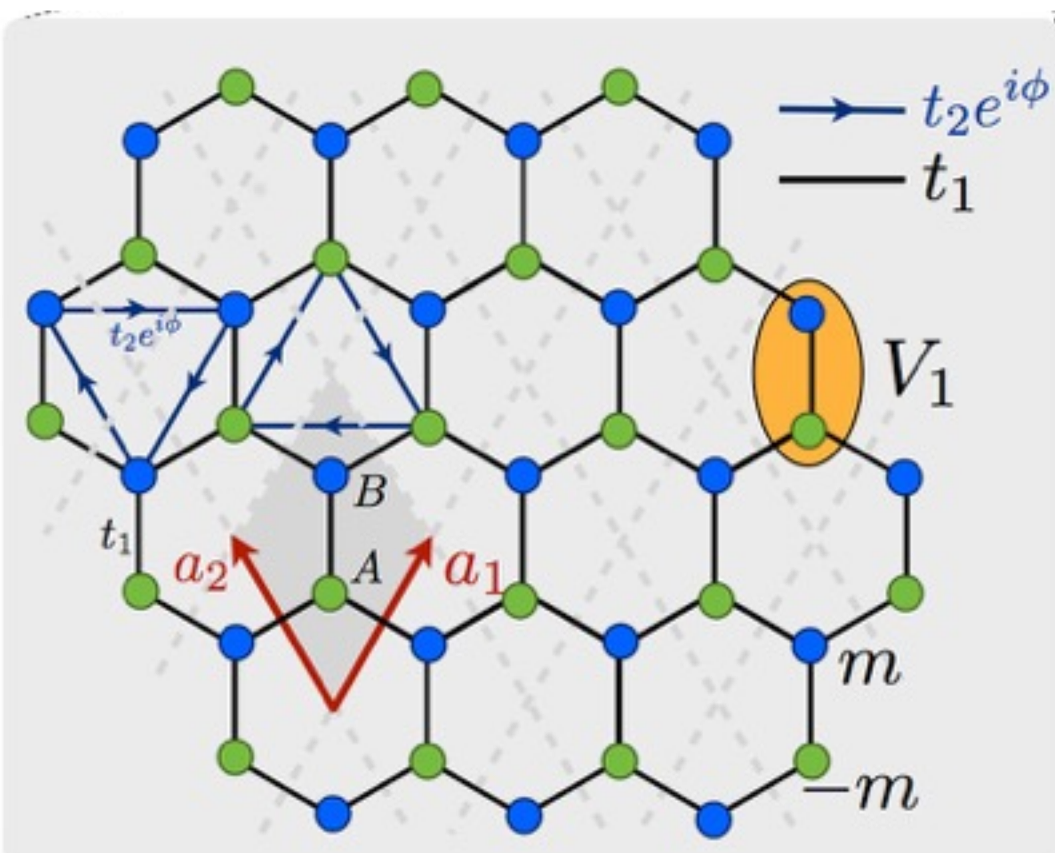
[Kitaev & Preskill '06]



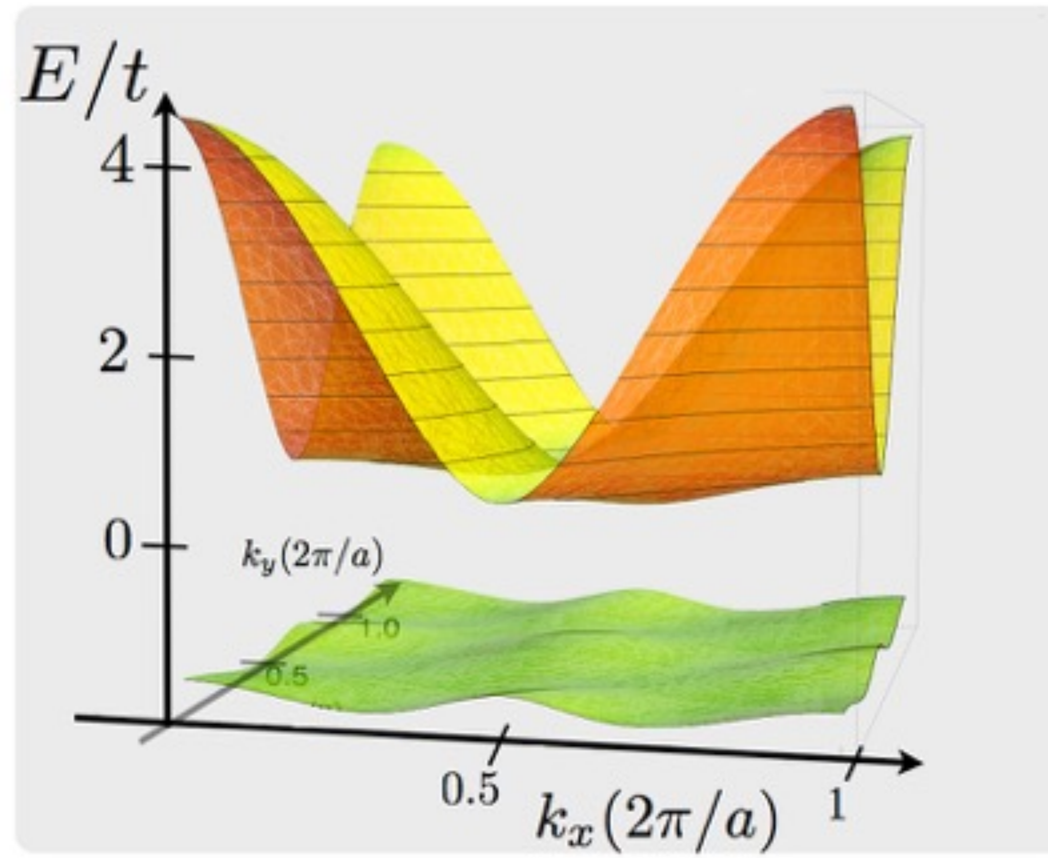


M. P. Zaletel, R. S. K. Mong, Z. Papic, FP, (in preparation).

Topological Order: Fractional Chern Insulators



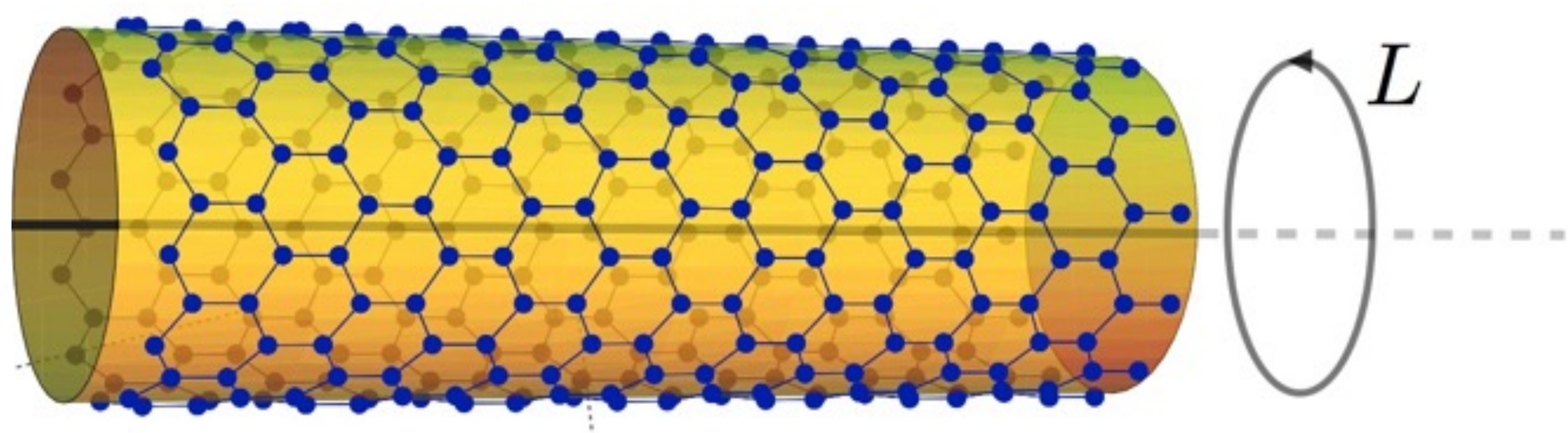
[Haldane ‘88]



[Neupert et al, Sun et al, Tang et al ’11]

Topological Order: Fractional Chern Insulators

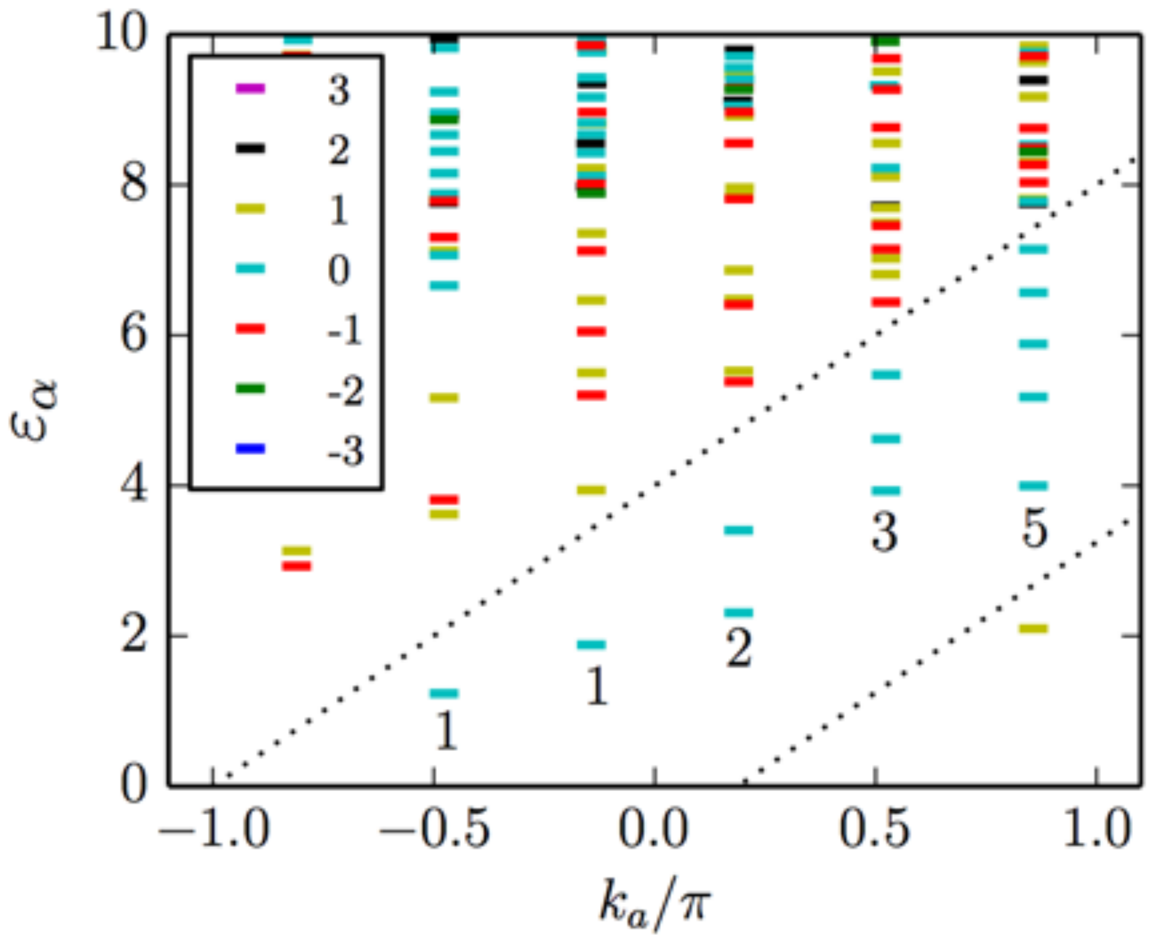
- **Density Matrix Renormalization Group (DMRG)** : **Fractional Chern Insulators**
  - $\nu = 1/3$ filling of the lowest band
  - Circumferences up to $L = 12$ sites



A. Grushin, J. Motruk, M. P. Zaletel, FP, Phys. Rev. B **91**, 035136 (2015).

Topological Order: Fractional Chern Insulators

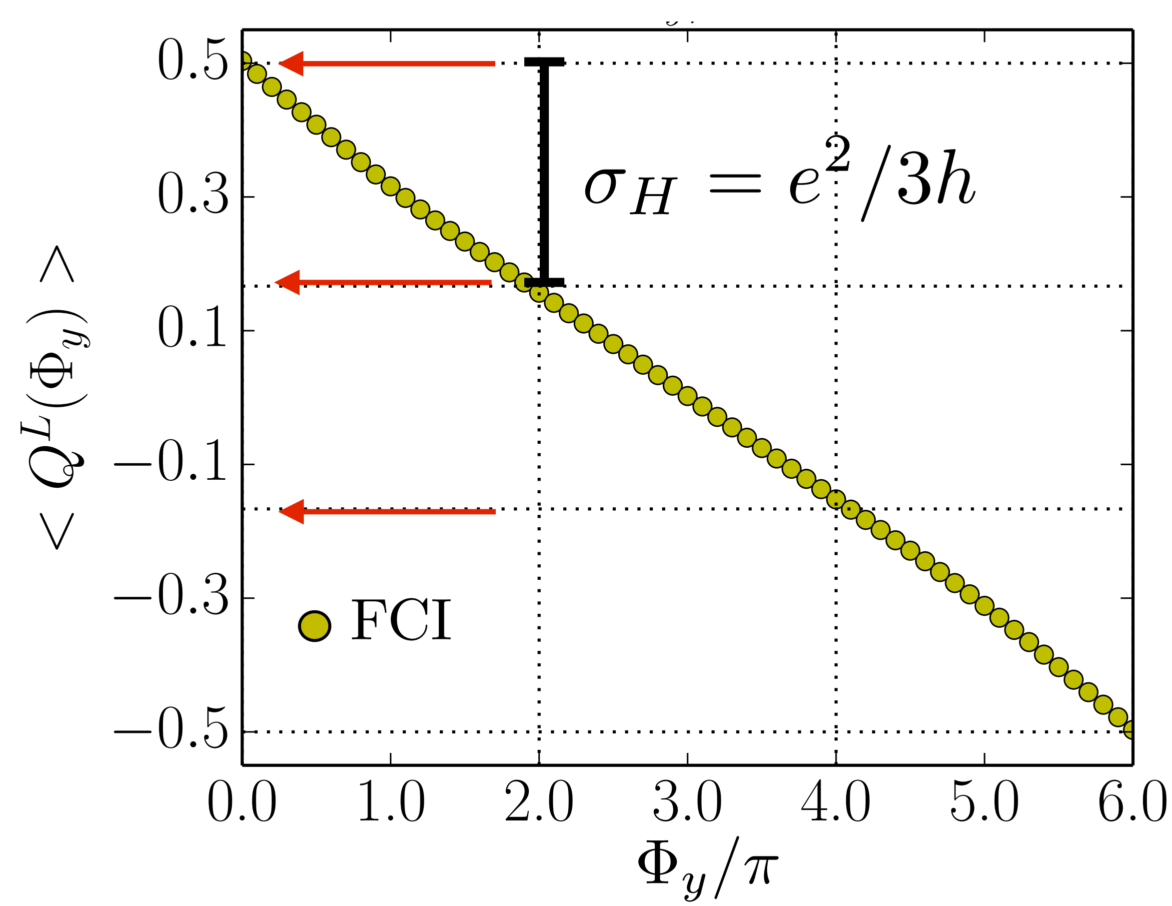
"CFT counting"

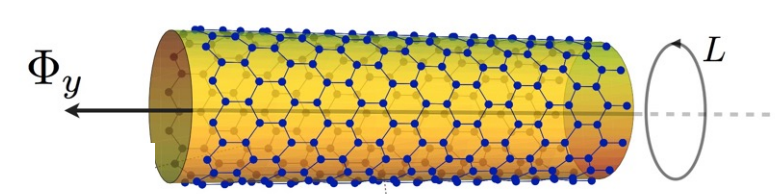


[Li & Haldane '08]

$$\rho^{\mathrm{red}} = \sum_\alpha \exp(-\epsilon_\alpha)|\phi_\alpha\rangle\langle\phi_\alpha|$$

"Charge pumping"

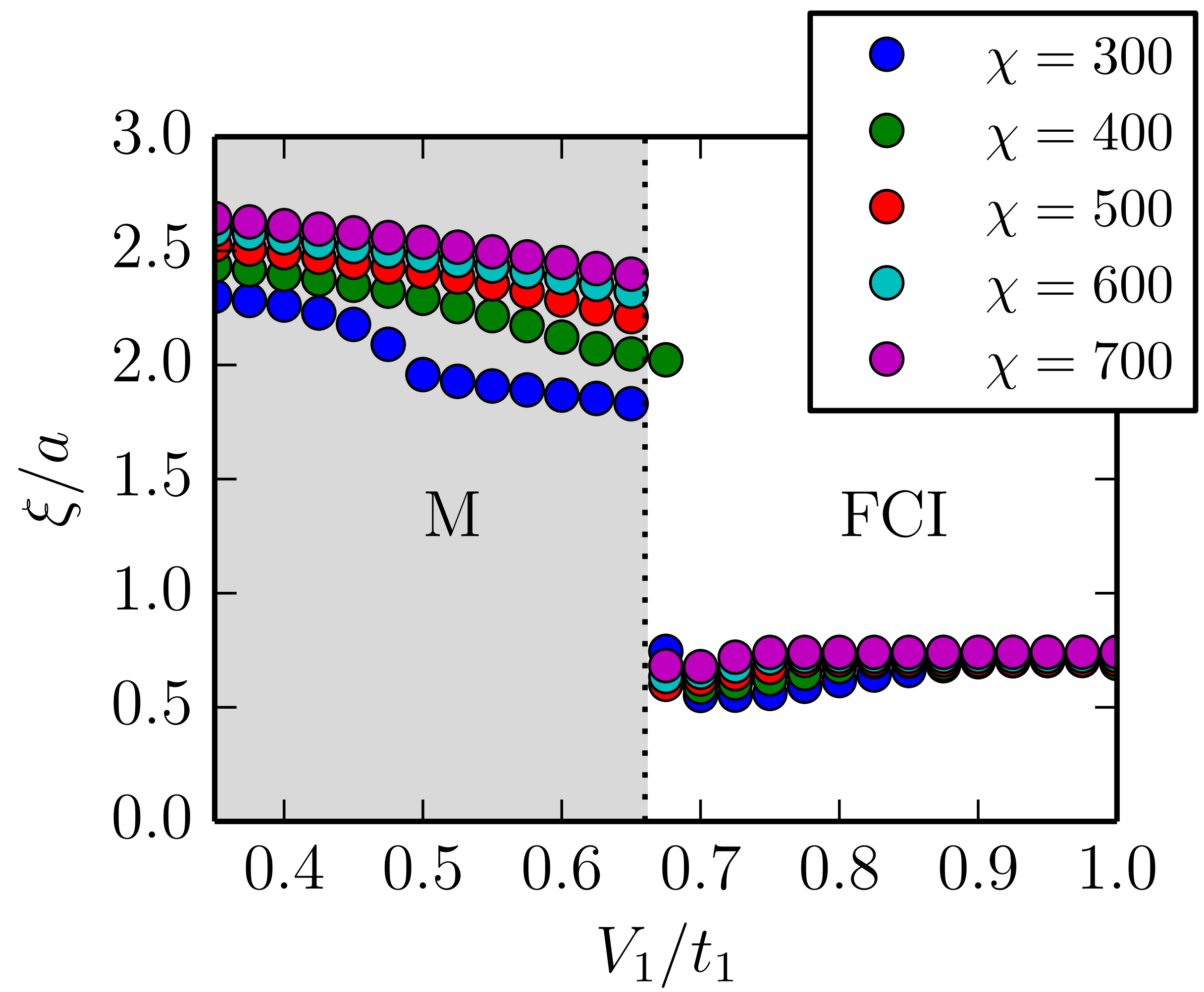




A. Grushin, J. Motruk, M. P. Zaletel, FP, Phys. Rev. B **91**, 035136 (2015).

Topological Order: Fractional Chern Insulators

- Evidence of the **first order metal to Fractional Chern Insulator phase transition!**

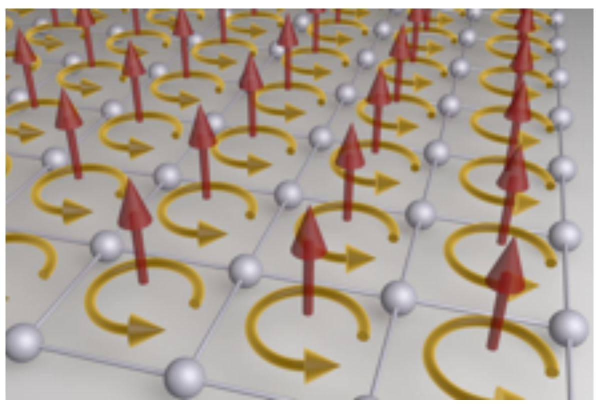


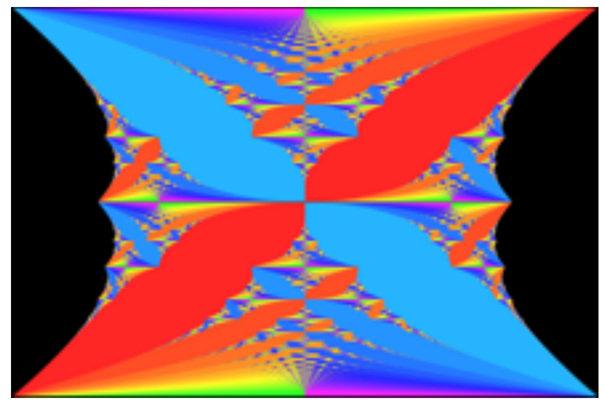
A. Grushin, J. Motruk, M. P. Zaletel, FP, Phys. Rev. B **91**, 035136 (2015).

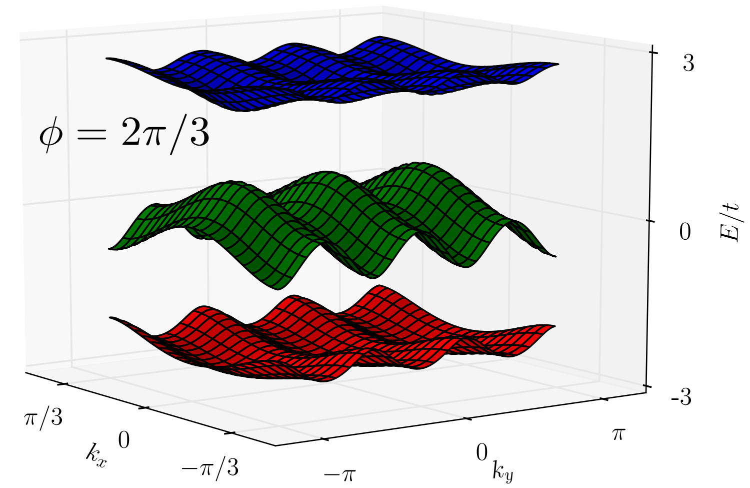
Fractionalized phases in the Hofstadter model

IN PROGRESS

- Artificial magnetic fields (Hofstadter model)
  [Bloch '14, Hofstadter '76]

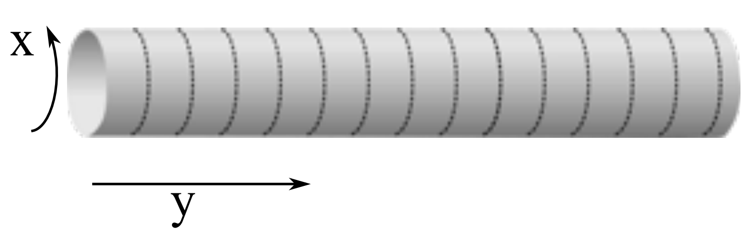


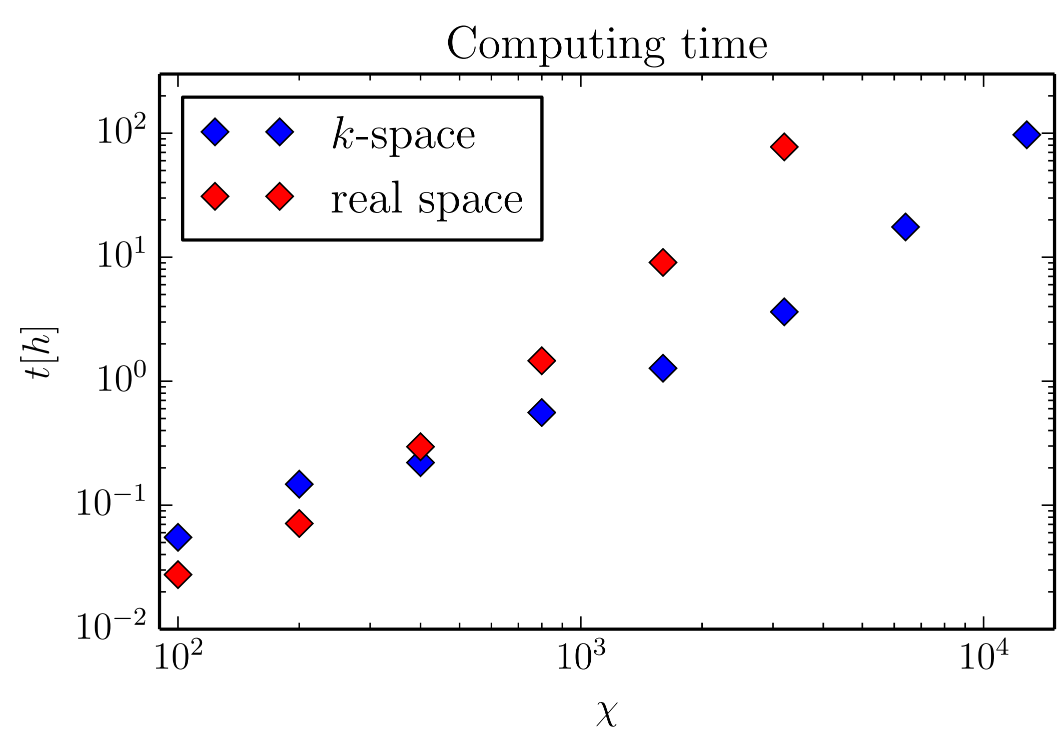




- iDMRG in mixed representations:

$$(x, y) \to (k_x, y)$$



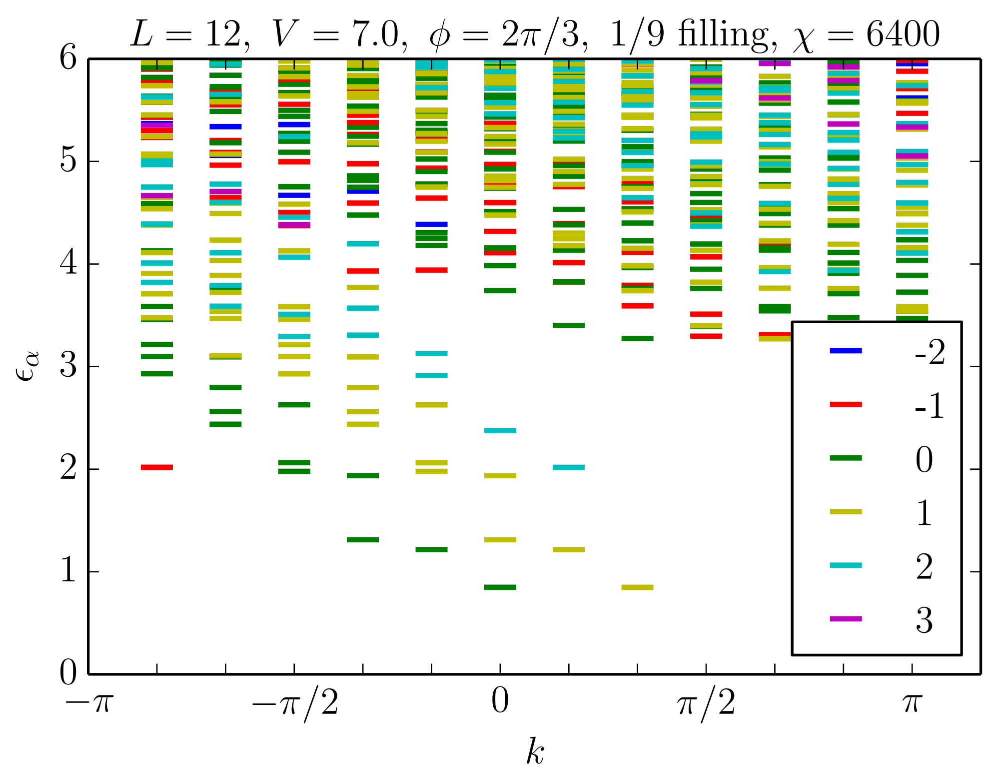


J. Motruk and FP (in progress)

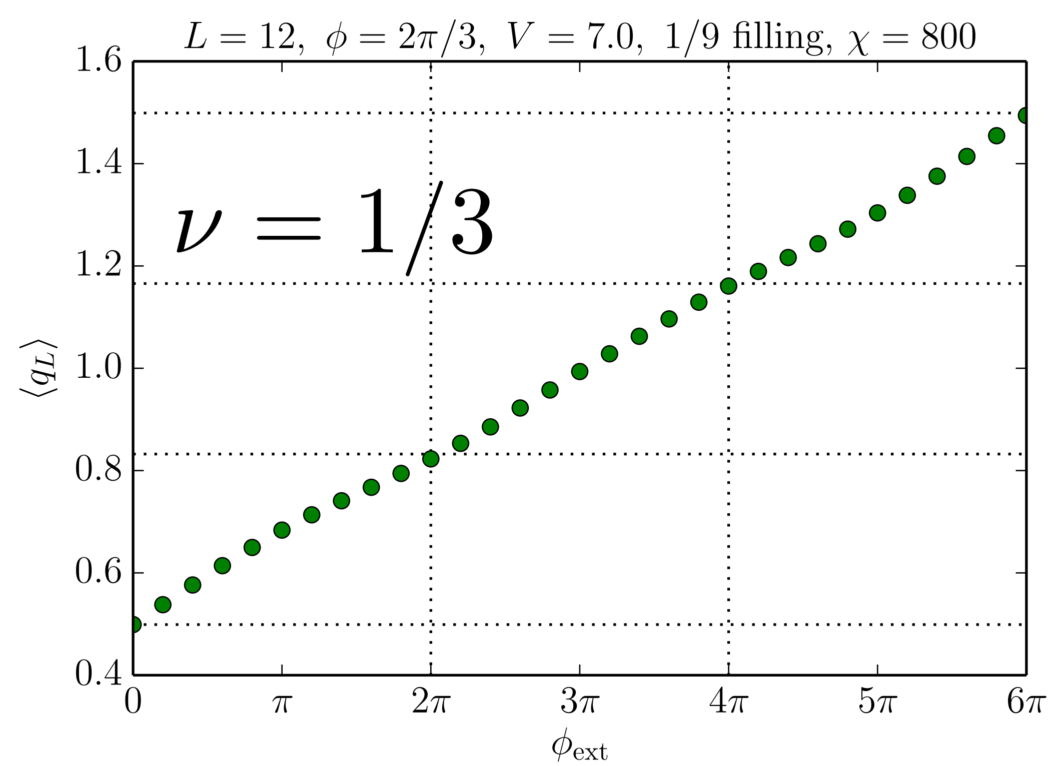
Fractionalized phases in the Hofstadter model

IN PROGRESS

"CFT counting"



"Charge pumping"



- Transitions between trivial and QH phases?
- Stability for experimentally relevant parameters?

J. Motruk and FP (in progress)

Summary

- Characterization of **intrinsic topologically ordered systems**
  - First numerical evidence for the existence of Fibonacci anyons in $\nu = 12/5$ FQH
  - Stability of an FCI phase in the Haldane model
  - Fractionalized phase in the Hofstadter model

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Johannes Motruk, MPIPKS


Mike Zaletel, Stanford
Roger Mong, Caltech


Thank You!