Phase diagram of spinless fermions on the honeycomb lattice

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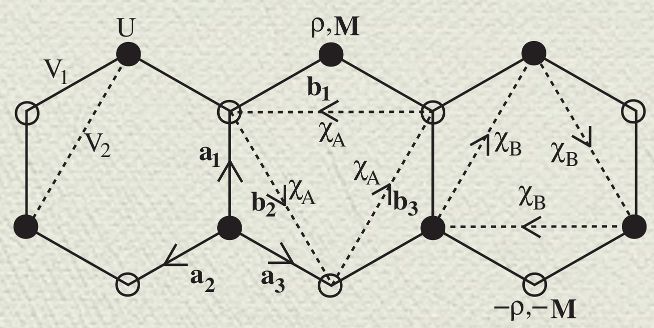
Ref: S. Capponi and A. M. Läuchli, in preparation

Motivation

- What if one could generate **topological insulating phases** without needing spin-orbit ?
- There are well-known examples of topological phases: insulators in the bulk but with gapless edge states

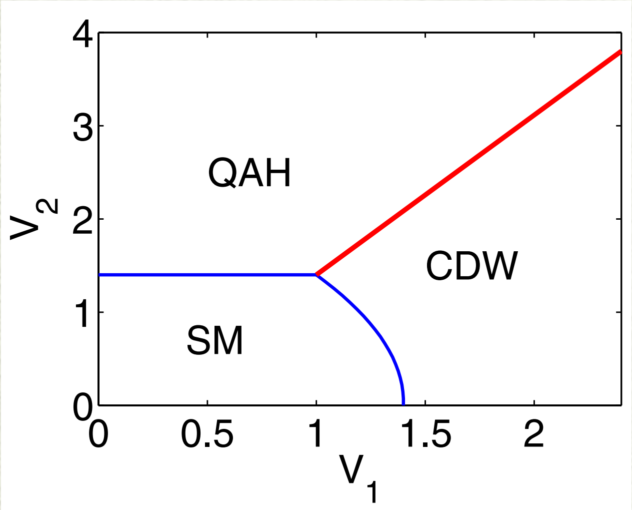
Quantum Anomalous Hall (QAH)	Quantum spin Hall (QSH)
spinless fermions	spinful fermions
breaks $\mathcal{T}$	does not break $\mathcal{T}$
chiral edge mode	helical edge mode
charge currents	spin currents
Haldane, PRL ‘88	see review by X.-L. Qi and S.-C. Zhang, RMP ‘11



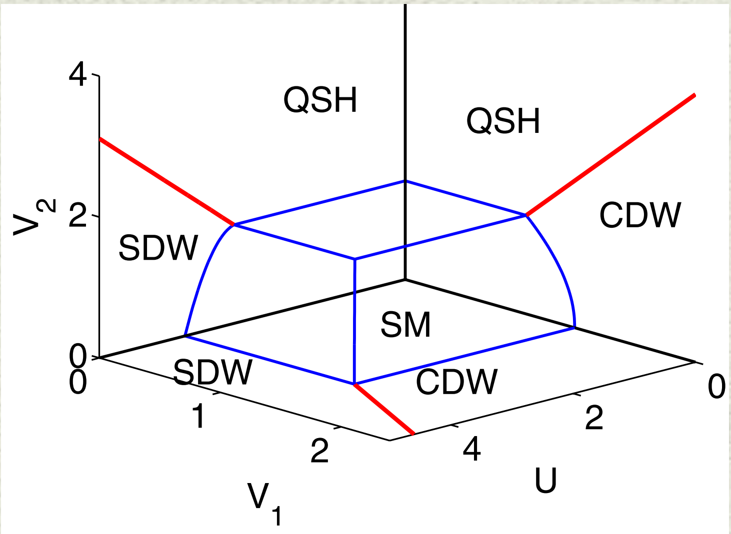
Motivation

- Mean-field results for simple model: YES !
- In the spinless case, this realizes famous Haldane model with interactions (aka Chern insulator) !

spinless fermions



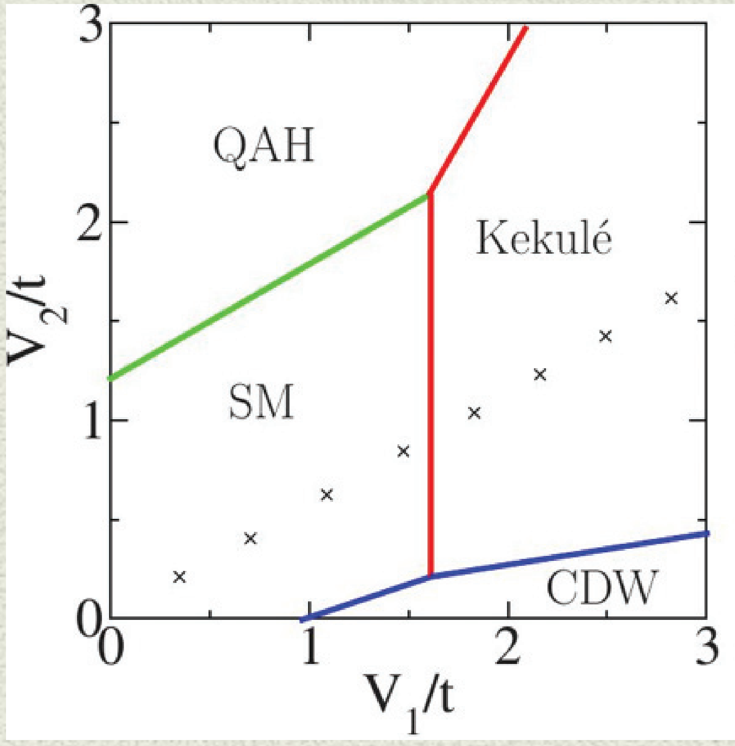
spinful fermions



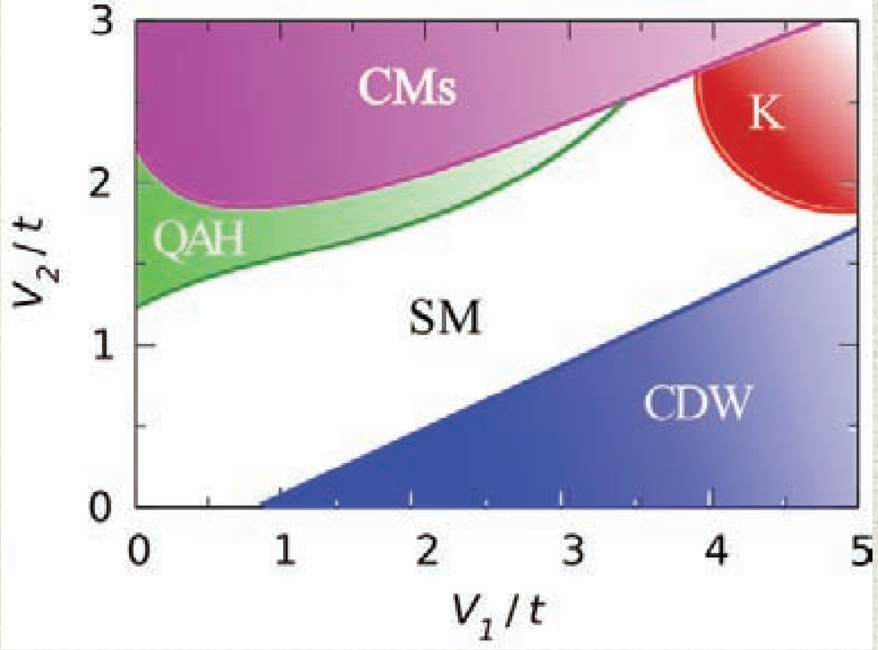
S. Raghu, X.-L. Qi, C. Honerkamp, S.-C. Zhang, PRL '08

What about classical phases ?

- But for large V's, one expects other CDW-like instabilities (Wigner crystallization) !
- This is indeed found using larger unit-cell mean-field approaches



C. Weeks and M. Franz, PRB '10



Grushin et al., PRB '13

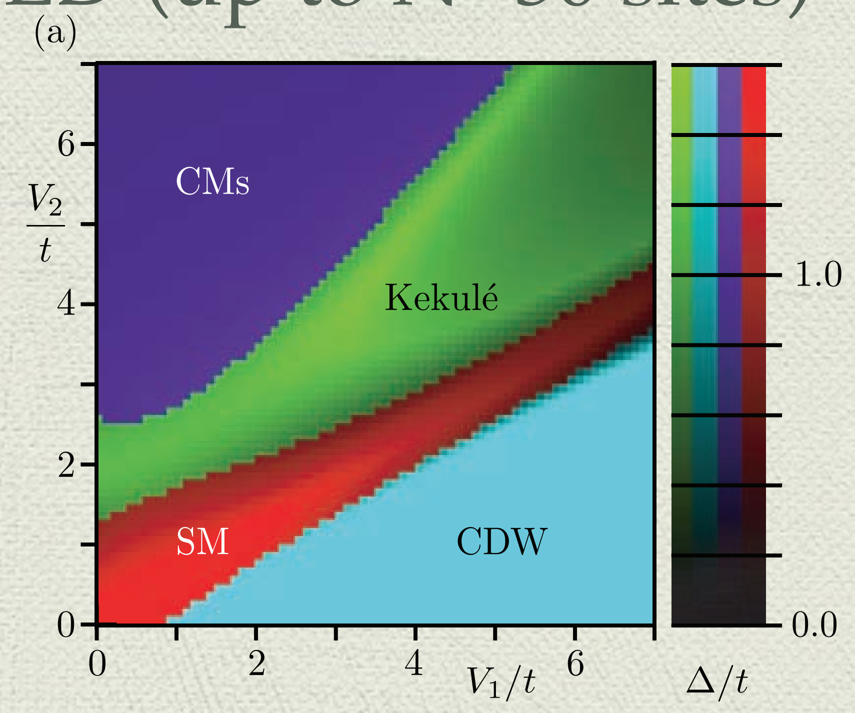
CM: charge modulation (breaks A <-> B)

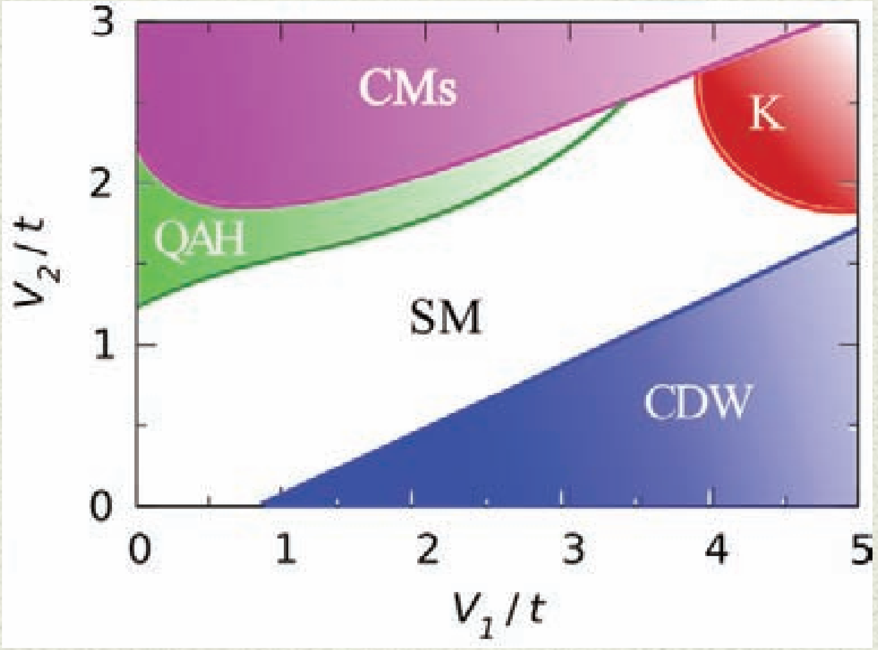
K = Kekulé pattern on hexagons

What about numerics ?

Hard problem since there is a sign problem (for V2>0) despite particle-hole symmetry

ED (up to N=30 sites)





García-Martínez et al., PRB ‘13

M. Daghofer and M. Hohenadler, PRB ‘14

Grushin et al., PRB ‘13

Outline

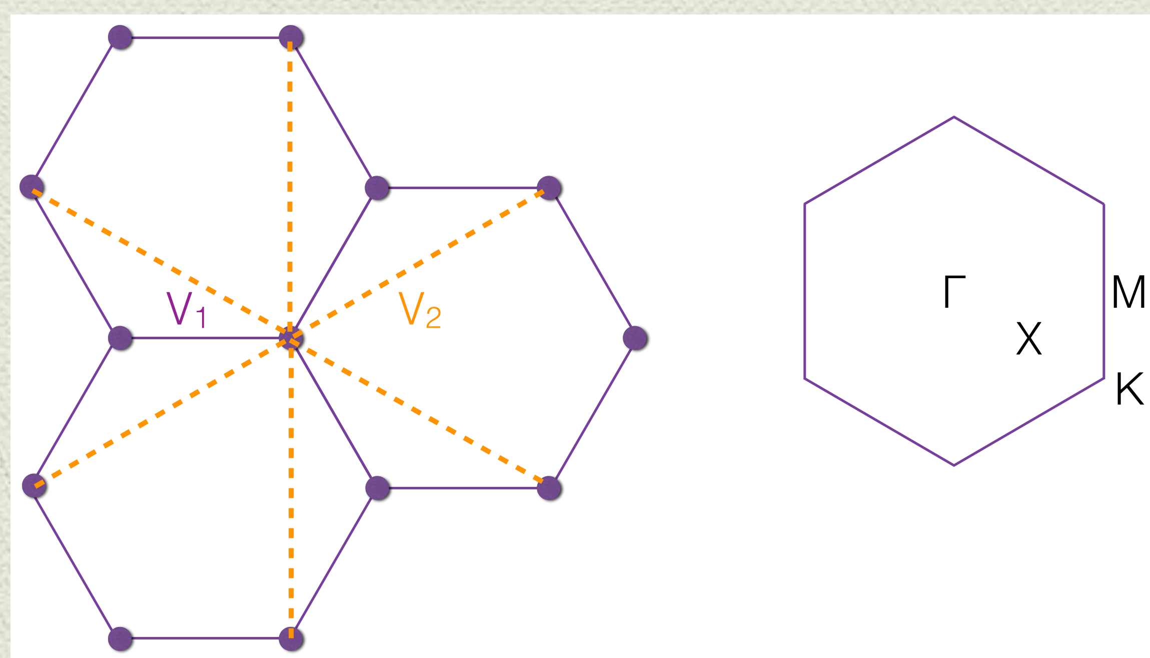
- Focus on the spinless case
- Study of the classical limits + perturbation
- Exact Diagonalization of the quantum model
- Global phase diagram and discussion

Model and methods

- Interacting spinless fermions on honeycomb

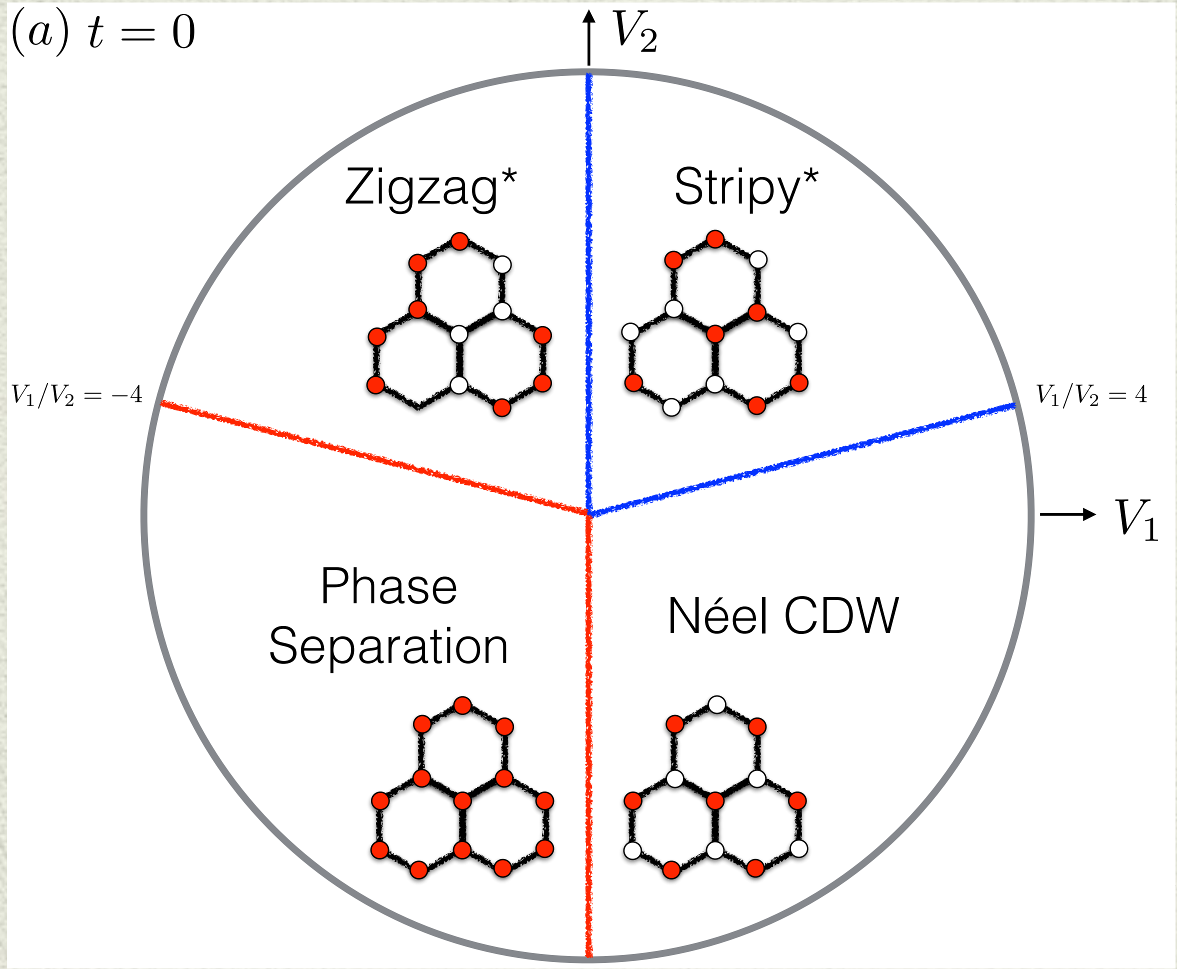
$$\mathcal{H} = -t \sum_{\langle ij \rangle} c_i^\dagger c_j + h.c. + V_1 \sum_{\langle ij \rangle} (n_i - 1/2)(n_j - 1/2) + V_2 \sum_{\langle\langle ij \rangle\rangle} (n_i - 1/2)(n_j - 1/2)$$

Working at half-filling



Limit of large interactions

- Ising model with competing interactions V1 and V2
- Then, we will ask what is the effect of a small t ?



Phase separation issue

Corboz et al., EPL '12

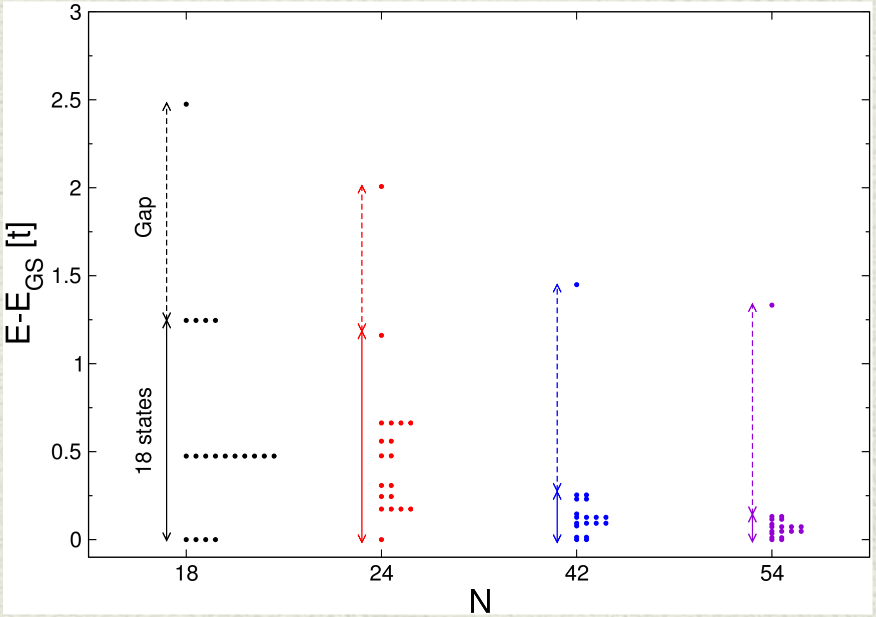
Limit of large interactions

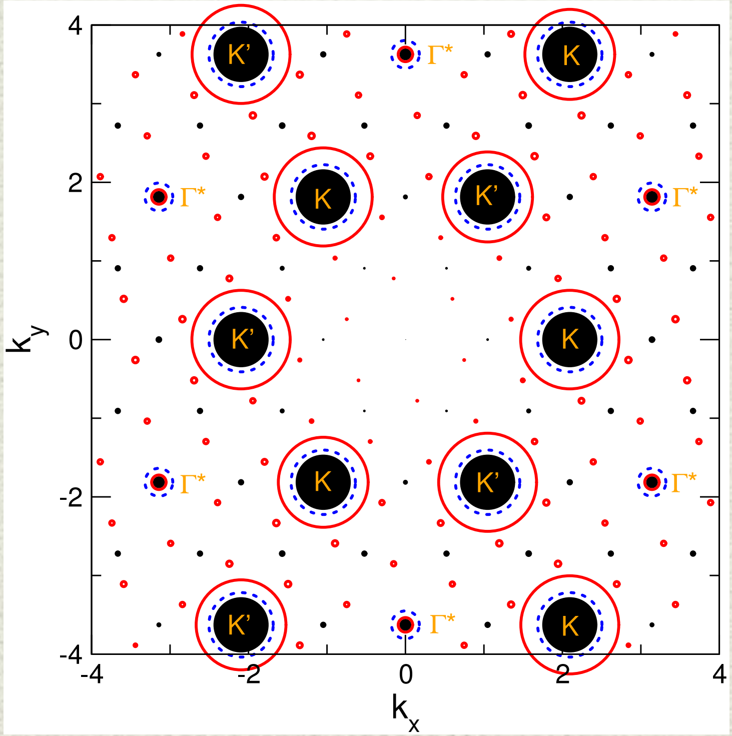
$V_2>0 \;\; V_1=0$

2 disconnected triangular lattices.
For each of them, any configuration with 1 or 2 particles per V_2 triangle is a GS.

extensive degeneracy at half-filling !
however **18** uud-ddu configurations are maximally flippable

effective model spectrum

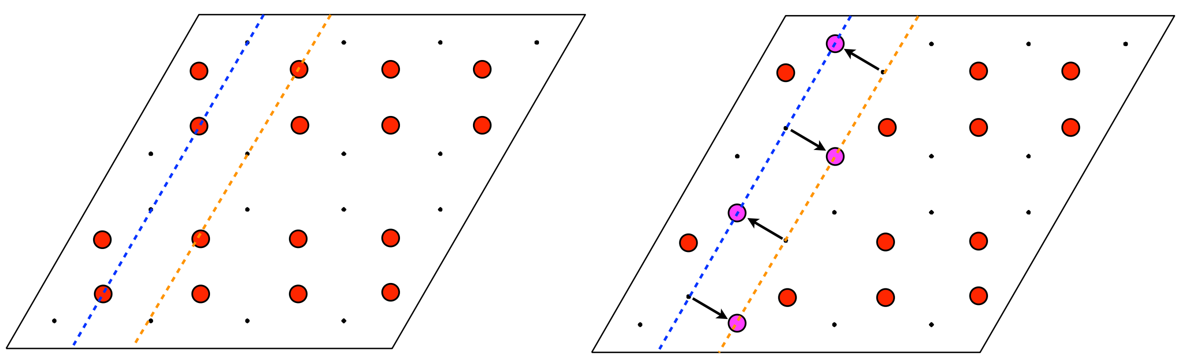




charge modulation + tripling of the unit cell

Limit of large interactions

$0<V_1<4V_2$ stripy states with sub-extensive degeneracy



$N = 32$

order-by-disorder should select some states !

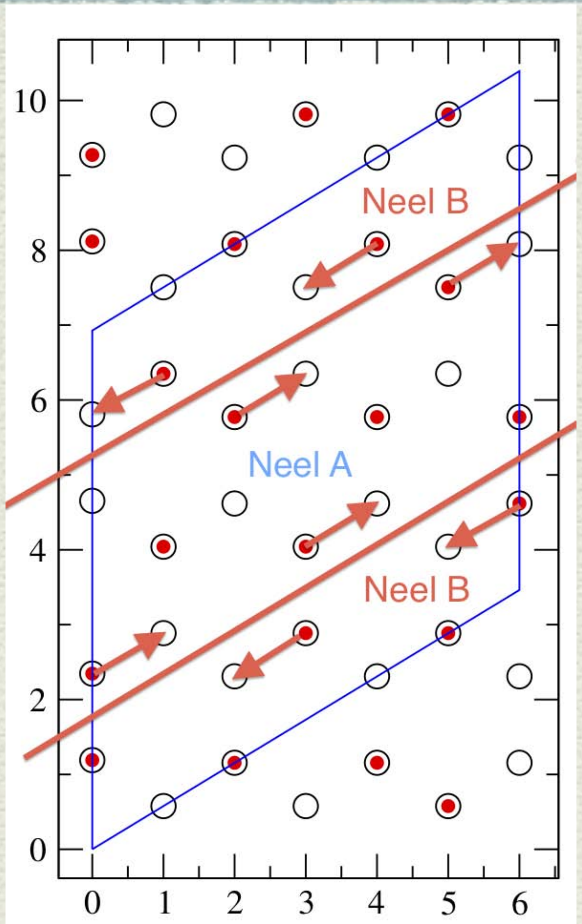
M point is crucial here

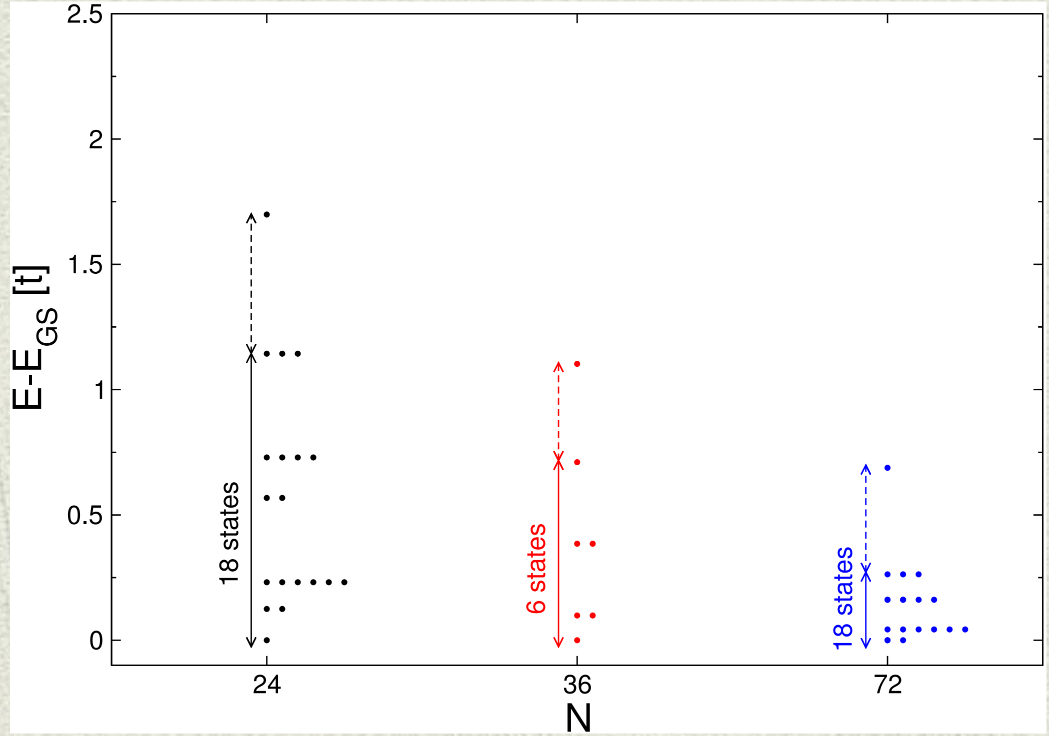
Limit of large interactions

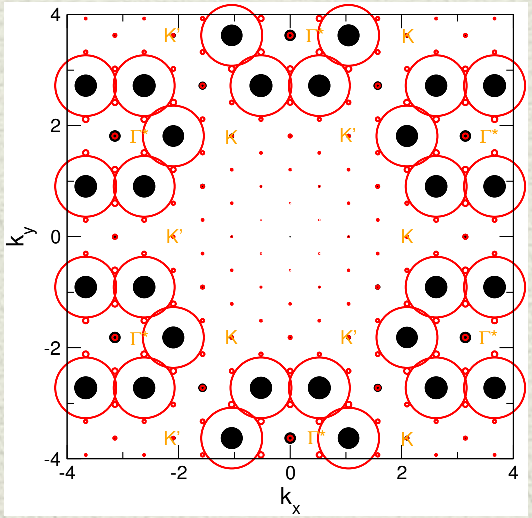
$V_1=4V_2$

extensive degeneracy

effective model selects Néel states with line defects (order by disorder):
there are 18 maximally flippable such states



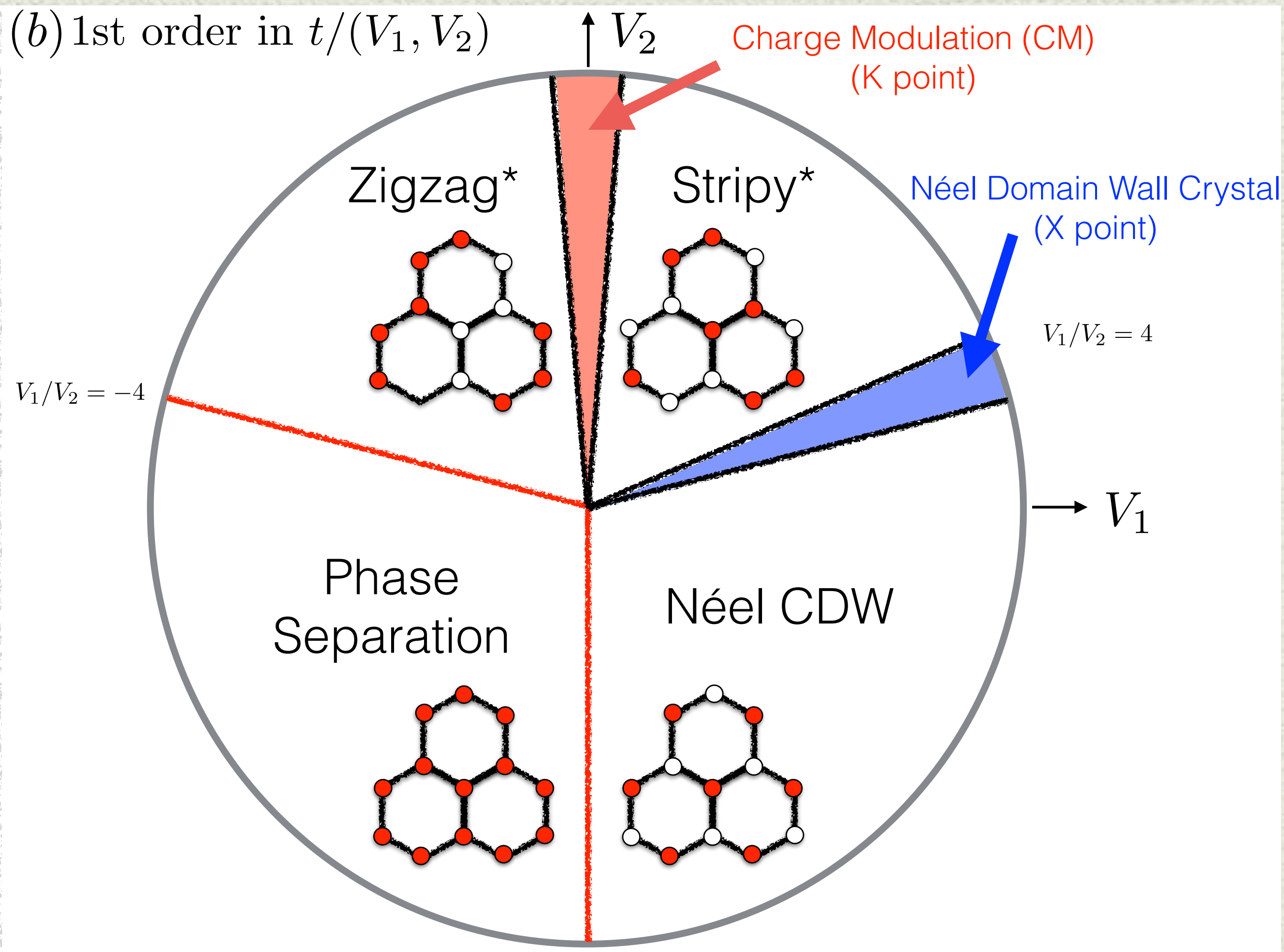




charge modulation + kinetic energy

Limit of large interactions

- Effect of a small hopping in perturbation

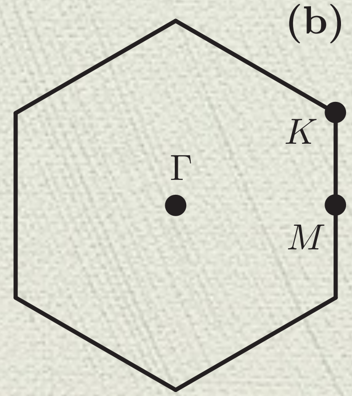


Exact Diagonalizations

- Cluster properties are crucial

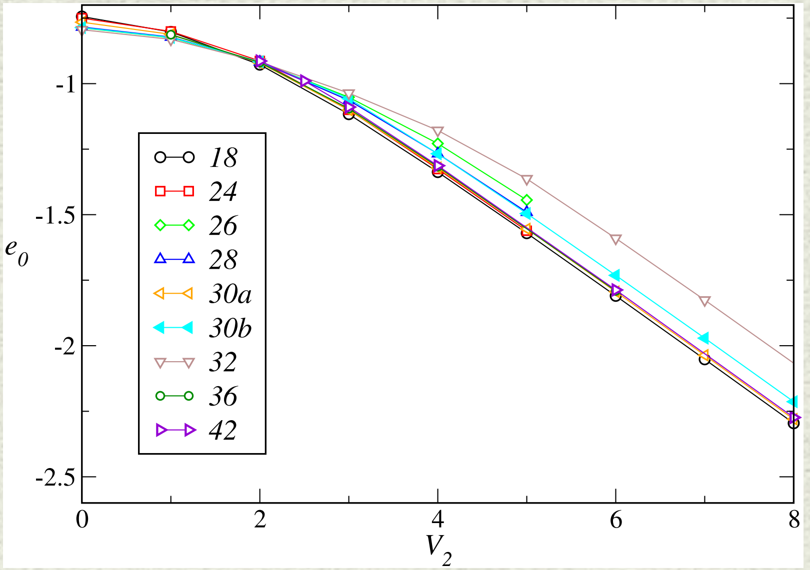
Name	**a**	**b**	sym. group	K (2)	M (3)	X (6)
18	$(6,0)$	$(3,3\sqrt{3})$	C_{6v}	**yes**	no	no
24	$(6,2\sqrt{3})$	$(0,4\sqrt{3})$	C_{6v}	**yes**	**yes**	**yes**
26	$(7,\sqrt{3})$	$(2,4\sqrt{3})$	C_6	no	no	no
28	$(7,\sqrt{3})$	$(0,4\sqrt{3})$	Z_2	no	**yes** (1)	no
30a	$(3,5\sqrt{3})$	$(-3,5\sqrt{3})$	$C_2 \times C_2$	**yes**	no	no
30b	$(5,3\sqrt{3})$	$(-5,3\sqrt{3})$	$C_2 \times C_2$	no	no	no
32	$(8,0)$	$(4,4\sqrt{3})$	C_{6v}	no	**yes**	no
34	$(9,\sqrt{3})$	$(2,4\sqrt{3})$	Z_2	no	no	no
36	$(6,0)$	$(6,6\sqrt{3})$	$C_2 \times C_2$	**yes**	**yes** (1)	**yes** (2)
38	$(8,2\sqrt{3})$	$(1,5\sqrt{3})$	C_6	no	no	no
40	$(10,0)$	$(4,4\sqrt{3})$	Z_2	no	**yes** (1)	no
42	$(9,\sqrt{3})$	$(3,5\sqrt{3})$	C_6	**yes**	no	no

(b)



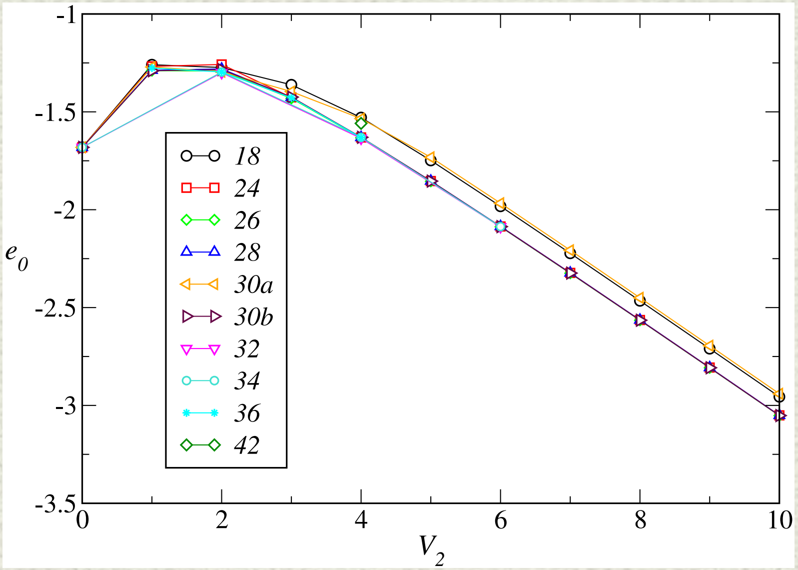
$V_1=0$

K point is important



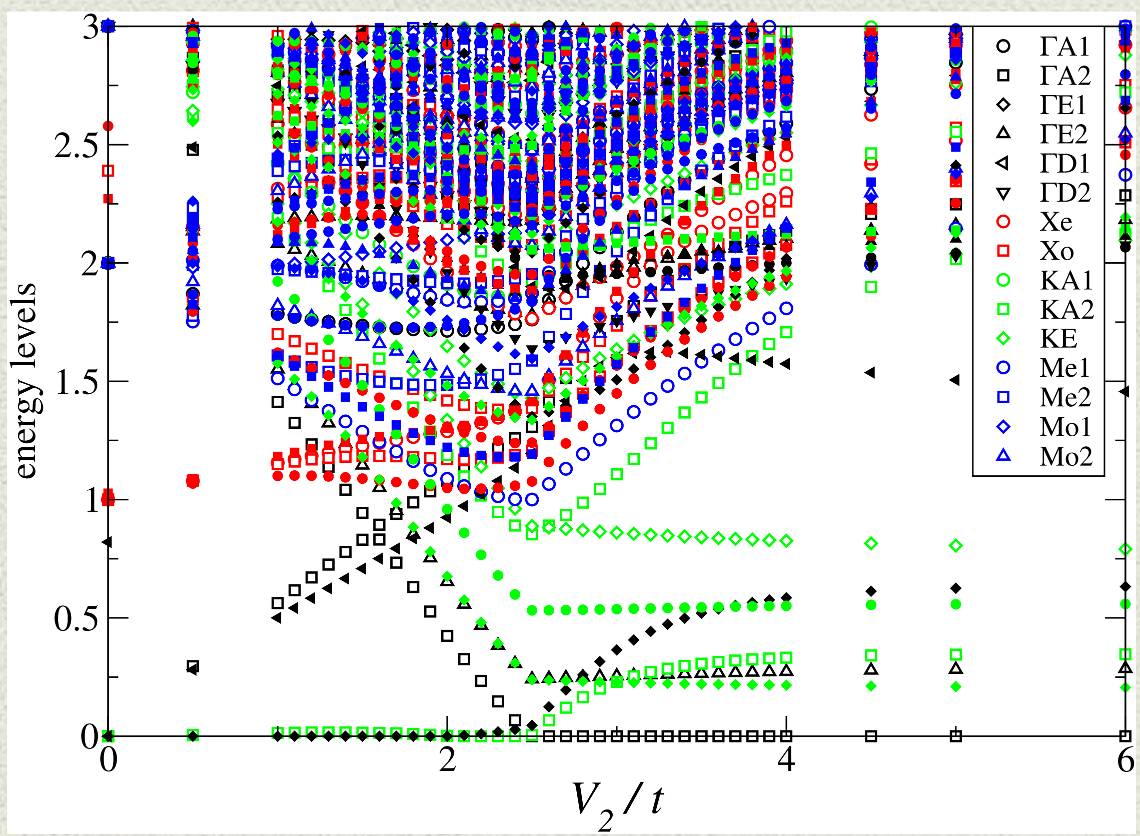
$V_1=4$

M point is important



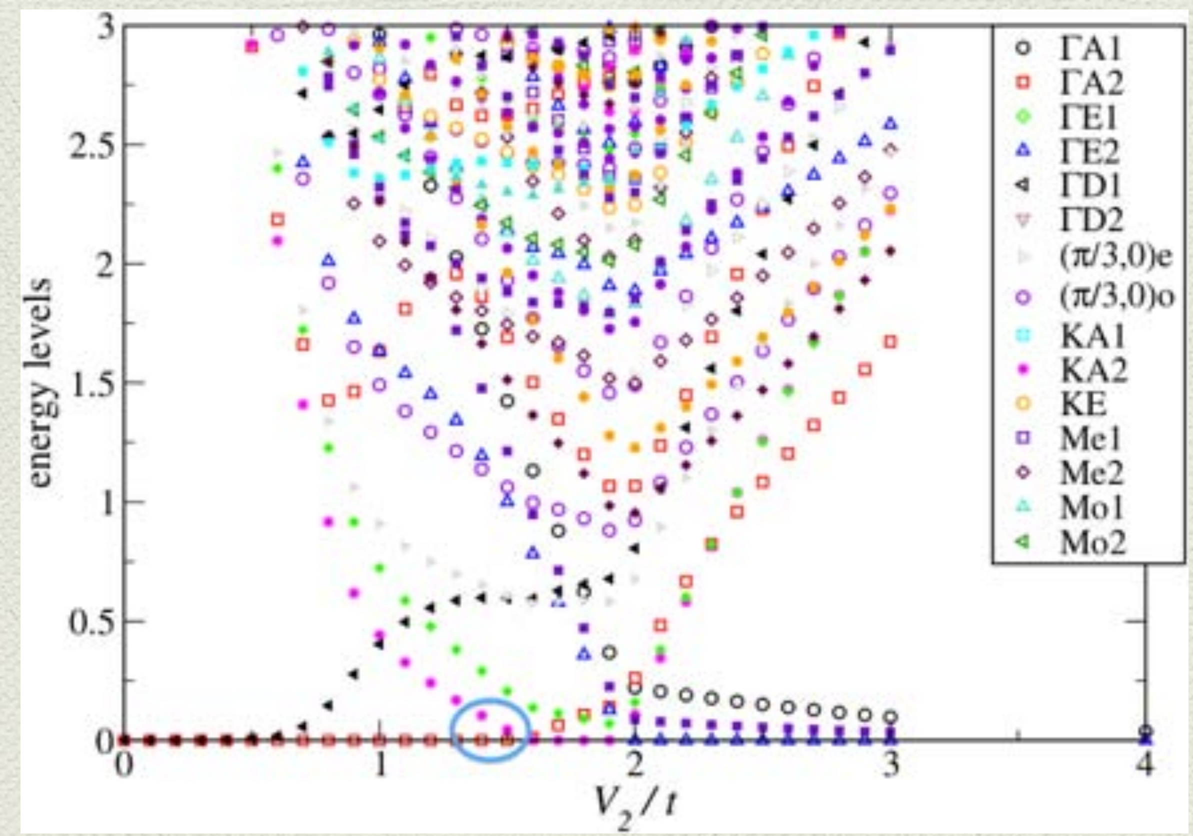
Energetics

V_1=0 line



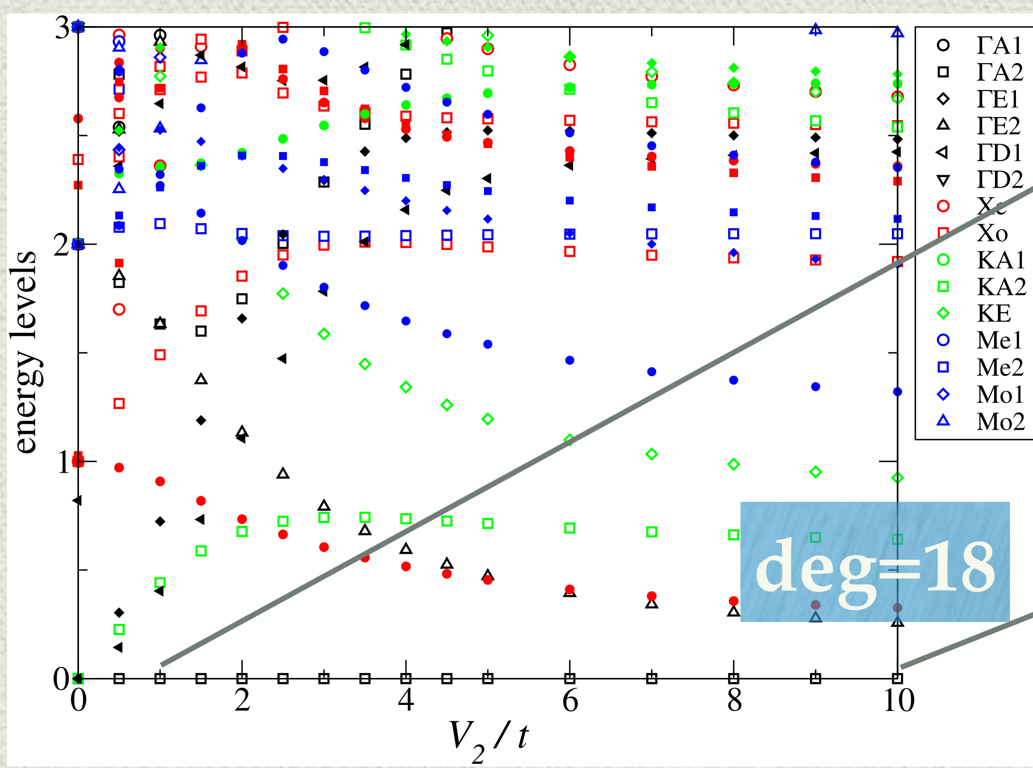
semi-metal charge-modulated deg=18

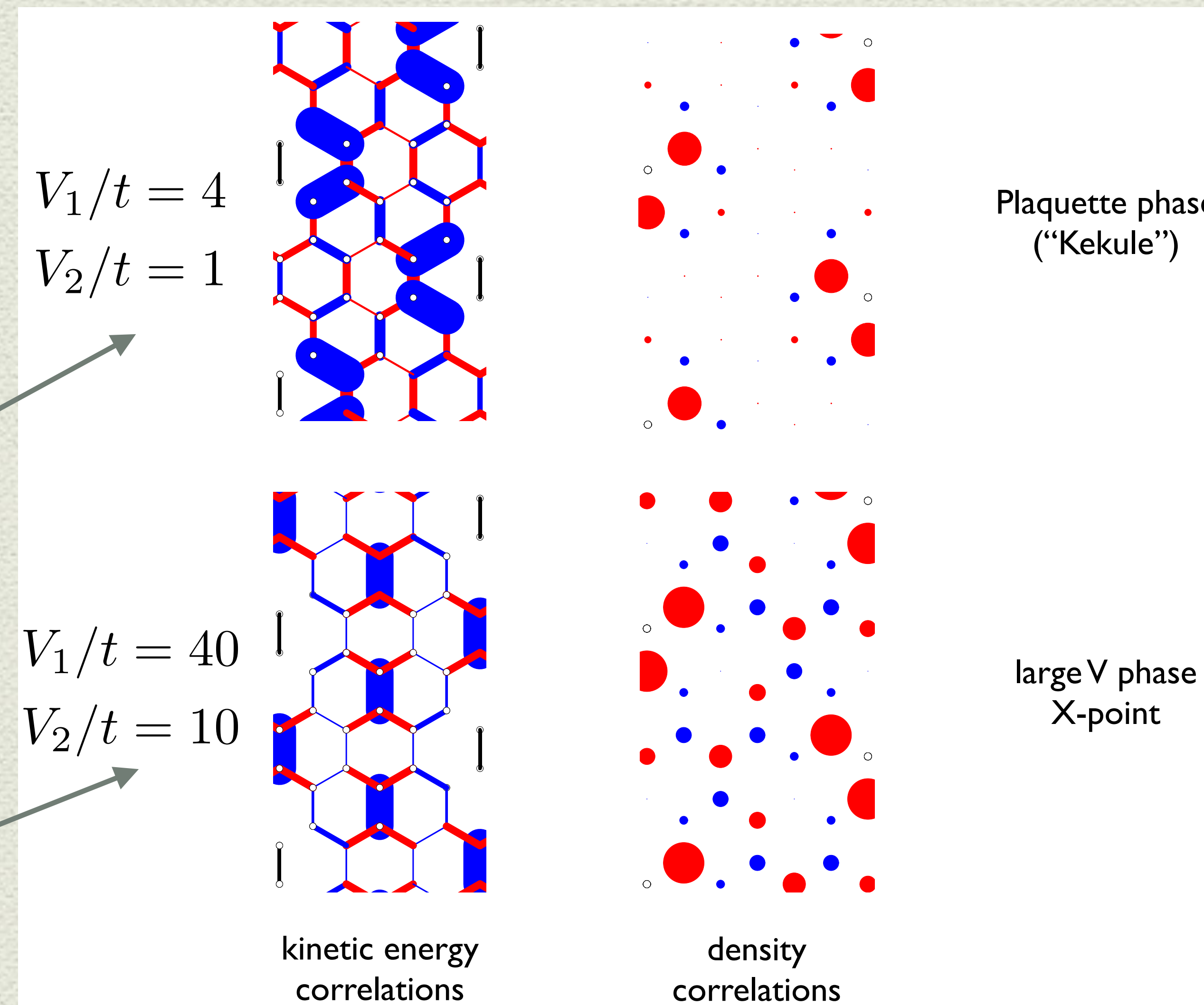
V_1=4 line



CDW Kekulé stripy

Along the $V_1 = 4V_2$ line





SM

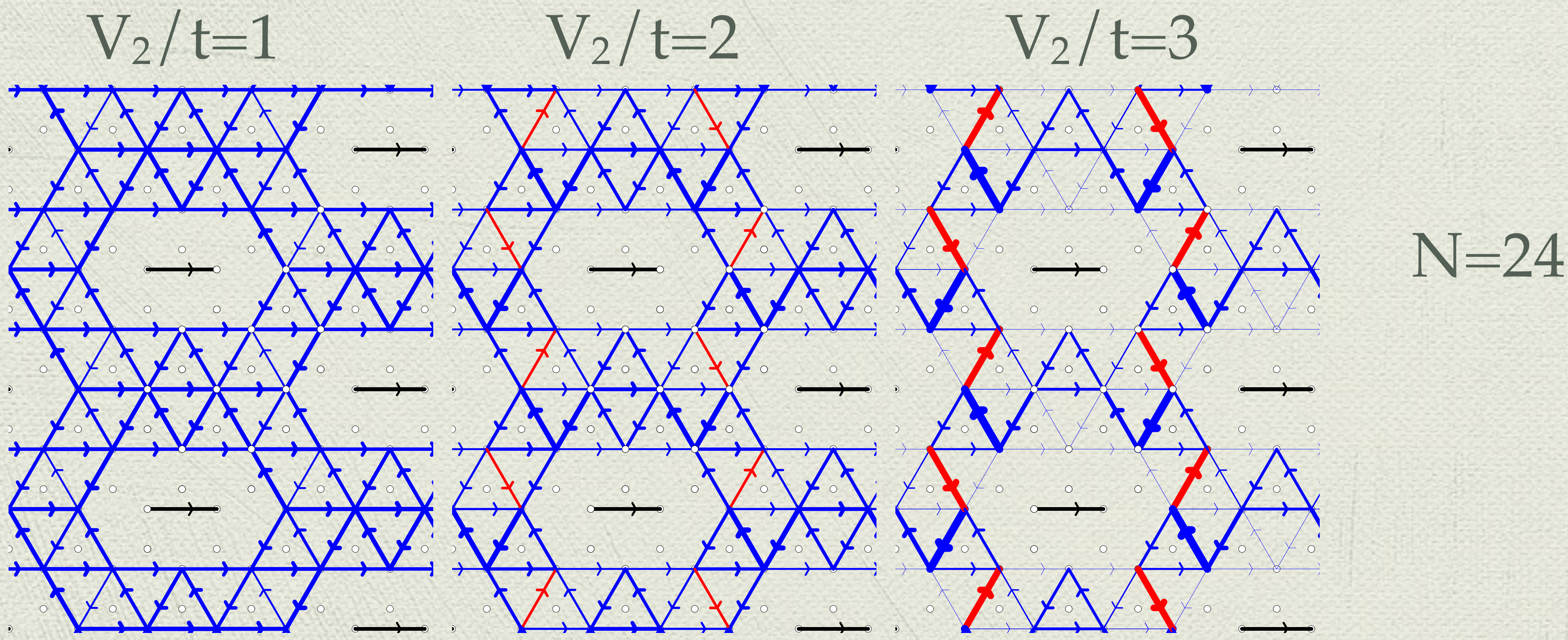
Kekulé

X phase

Evidence of a quantum phase transition

(Charge) current correlations

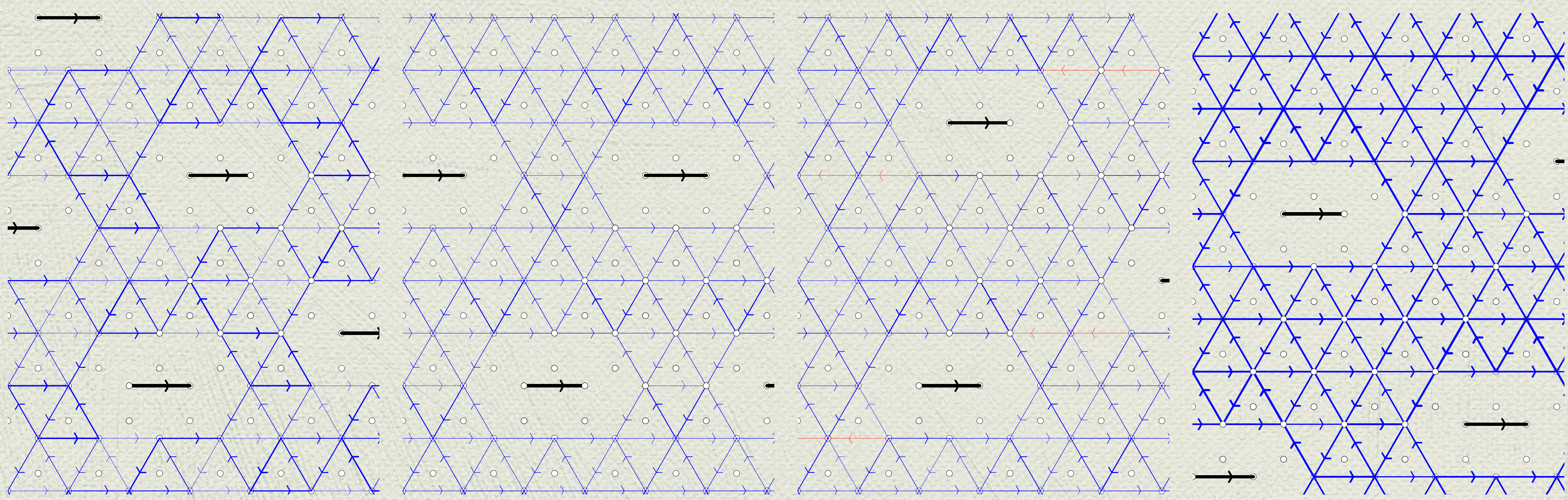
- For V_1=0, there exists a perfect correlation pattern only at small V_2/t



Finite-size scaling

- Sample-to-sample variations, but perfect correlations still

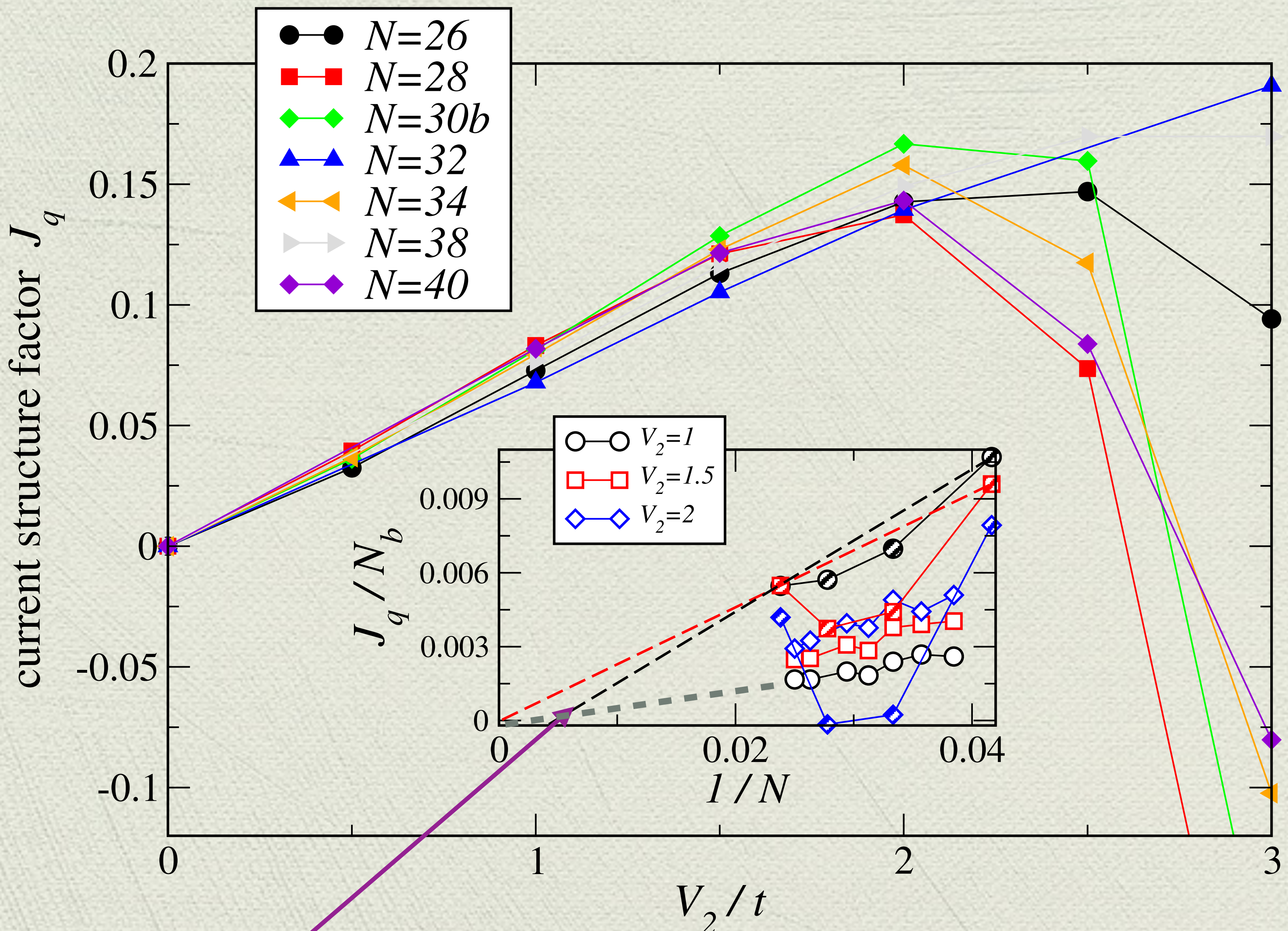
N=26 N=32 N=38 N=42



$V_2/t=1$ all these clusters have C6 symmetry

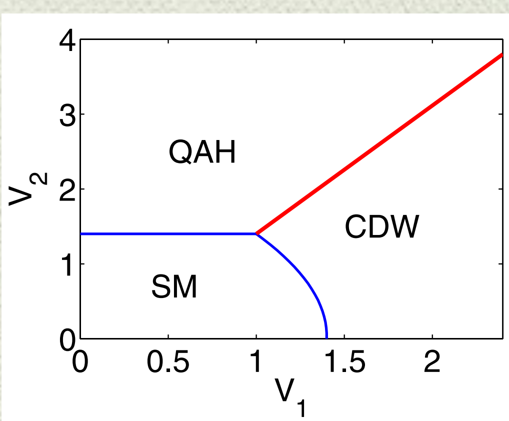
Current structure factor

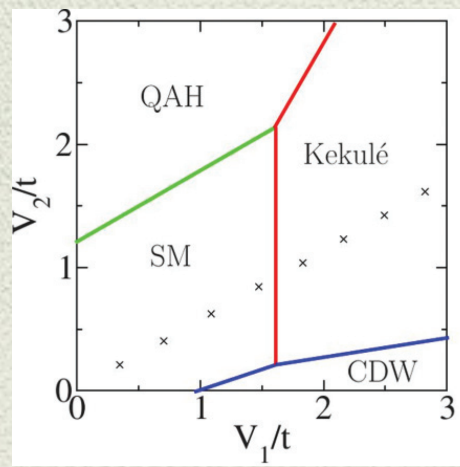
- Sample variations can be partly understood

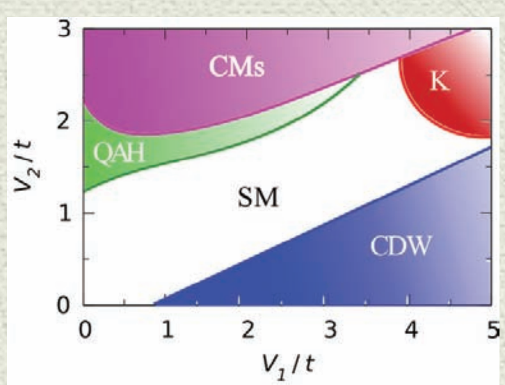


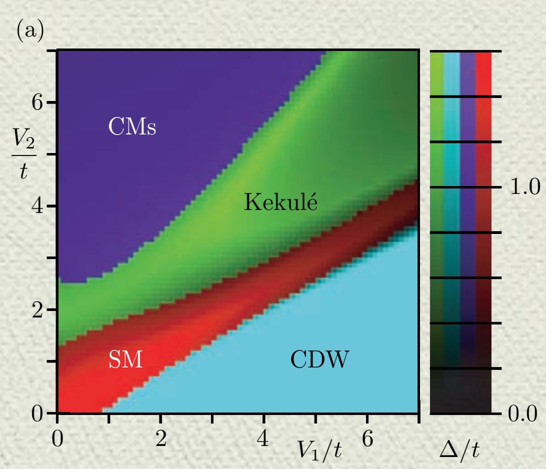
vanishing value !?

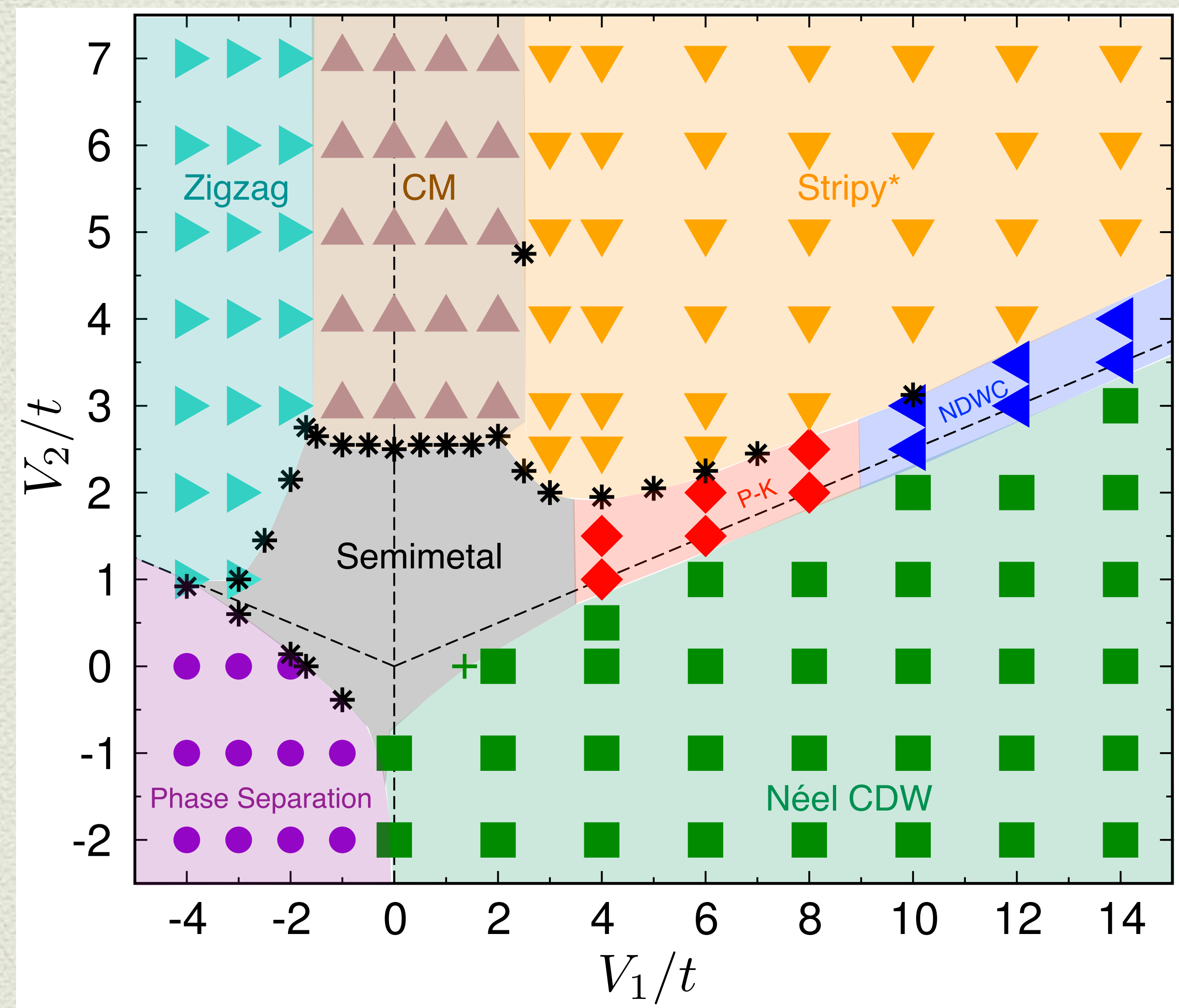
Global Phase Diagram











Concluding remarks

- Clarification of the phase diagram with new phases at large interactions
- No evidence for Chern insulators

Ref: S. Capponi and A. M. Läuchli, in preparation

Perspectives

- Possible progress in solving the sign problem, as was done for V_2=0 recently ?

E. Fulton Huffman and S. Chandrasekharan, PRB ‘14

L. Wang, P. Corboz and M. Troyer, NJP ‘14

Z.-X. Li, Y.-F. Jiang, and H. Yao, arXiv:1408

- Investigate closely related models to look for topological insulators !